Biostatistics 615/815
Implementing Algorithms in C++

Hyun Min Kang

January 11th, 2011
Recap from the Last Lecture

- **Algorithm**: A sequence of computational steps from input to output
  - SingOldMacDonaldSong
  - InsertionSort
  - TowerOfHanoi

- **Implementation in C++**
  - helloWorld
  - towerOfHanoi
  - insertionSort (skipped)
Recap - hellolWorld

Writing hellolWorld.cpp

```cpp
#include <iostream> // import input/output handling library

int main(int argc, char** argv) {
    std::cout << "Hello, World" << std::endl;
    return 0; // program exits normally
}
```

Compiling hellolWorld.cpp

Install Cygwin (Windows), Xcode (MacOS), or nothing (Linux).

```
user@host:~/$ g++ -o helloWorld helloWorld.cpp
```

Running hellolWorld

```
user@host:~/$ ./helloWorld
Hello, World
```
**How helloWorld works**

**main()**: First function to be called

```c
// type of return value is integer
// return value of main() function is program exit code
// 0 is normal exit code and the others are abnormal exit codes
int main()
{
    // function arguments are surrounded by parentheses
    int argc, // number of command line arguments
    char** argv // list of command line arguments - will explain later

    // ... function body goes here
    return 0; // return normal exit code
}
```

// name (identifier) of function is 'main'
// 'main' is a special function, invoked at the beginning of a program.

// type of return value is integer
// return value of main() function is program exit code
// 0 is normal exit code and the others are abnormal exit codes
int main()
{
    // function arguments are surrounded by parentheses
    int argc, // number of command line arguments
    char** argv // list of command line arguments - will explain later

    // ... function body goes here
    return 0; // return normal exit code
}
How `helloWorld` works

Using `iostream` to output strings to console

```cpp
// includes standard library for handling I/Os (inputs/outputs)
// std::cout and std::endl cannot be recognized without including <iostream>
#include <iostream>

int main (int argc, char** argv) {
    std::cout // standard output stream - messages are printed to console.
        << // insertion operator : appends the next value to the output stream
        "Hello, World" // string appended to std::cout via operator<<
        << // insertion operator : appends the next value to the output stream
        std::endl; // end-of-line appended to std::cout via operator<<
    return 0;
}
```
Today’s Lecture

What to expect

- Basic Data Types
- Control Structures
- Pointers and Functions
Today’s Lecture

What to expect

- Basic Data Types
- Control Structures
- Pointers and Functions

What are expected for the next few weeks

- The class does NOT focus on teaching programming language itself
- Expect to spend time to be familiar to programming languages
- VERY important to practice writing code on your own.
- Utilize office hours or after-class minutes for detailed questions in practice
Declaring Variables

Variable Declaration and Assignment

```
int foo; // declare a variable
foo = 5; // assign a value to a variable.
int foo = 5; // declaration + assignment
```
Declaring Variables

Variable Declaration and Assignment

```c
int foo; // declare a variable
foo = 5; // assign a value to a variable.
int foo = 5; // declaration + assignment
```

Variable Names

```c
int poodle; // valid
int Poodle; // valid and distinct from poodle
int my_stars3; // valid to include underscores and digits
int 4ever; // invalid because it starts with a digit
int double; // invalid because double is C++ keyword
int honky-tonk; // invalid -- no hyphens allowed
```
Data Types

### Signed Integer Types

```c
short foo; // 16 bits (2 bytes): -32,768 <= foo <= 32,767
int foo;   // Mostly 32 bits (4 bytes): -2,147,483,648 <= foo <= 2,147,483,647
long foo;  // 32 bits (4 bytes): -2,147,483,648 <= foo <= 2,147,483,647
long long foo; // 64 bits
short foo = 0;
foo = foo - 1;   // foo is -1
```
Data Types

Signed Integer Types

```plaintext
short foo; // 16 bits (2 bytes) : -32,768 <= foo <= 32,767
int foo;   // Mostly 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long foo;  // 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long long foo; // 64 bits
short foo = 0;
foo = foo - 1;  // foo is -1
```

Unsigned Integer Types

```plaintext
unsigned short foo; // 16 bits (2 bytes) : 0 <= foo <= 65,535
unsigned int foo;  // Mostly 32 bits (4 bytes) : 0 <= foo <= 4,294,967,295
unsigned long foo; // 32 bits (4 bytes) : 0 <= foo <= 4,294,967,295
unsigned long long foo; // 64 bits
unsigned short foo = 0;
foo = foo - 1;    // foo is 65,535
```
## Floating Point Numbers

### Comparisons

<table>
<thead>
<tr>
<th>Type</th>
<th>float</th>
<th>double</th>
<th>long double</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision</td>
<td>Single</td>
<td>Double</td>
<td>Quadruple</td>
</tr>
<tr>
<td>Size (in most modern OS)</td>
<td>32 bits</td>
<td>64 bits</td>
<td>128 bits</td>
</tr>
<tr>
<td>Sign</td>
<td>1 bit</td>
<td>1 bit</td>
<td>1 bit</td>
</tr>
<tr>
<td>Exponent</td>
<td>8 bits</td>
<td>11 bits</td>
<td>15 bits</td>
</tr>
<tr>
<td>Fraction (decimal digits)</td>
<td>23 bits</td>
<td>52 bits</td>
<td>112 bits</td>
</tr>
<tr>
<td>Minimum (&gt;0)</td>
<td>$1.2 \times 10^{-38}$</td>
<td>$2.2 \times 10^{-308}$</td>
<td>$3.4 \times 10^{-4932}$</td>
</tr>
<tr>
<td>Maximum</td>
<td>$3.4 \times 10^{38}$</td>
<td>$1.8 \times 10^{308}$</td>
<td>$1.2 \times 10^{4932}$</td>
</tr>
</tbody>
</table>
Handling Floating Point Precision Carefully

```
#include <iostream>

int main(int argc, char** argv) {
    float smallFloat = 1e-8; // a small value
    float largeFloat = 1.;    // difference in 8 (>7.2) decimal figures.
    std::cout << smallFloat << std::endl; // "1e-08" is printed
    smallFloat = smallFloat + largeFloat; // smallFloat becomes exactly 1
    smallFloat = smallFloat - largeFloat; // smallFloat becomes exactly 0
    std::cout << smallFloat << std::endl; // "0" is printed
    // similar thing happens for doubles (e.g. 1e-20 vs 1).
    return 0;
}
```
Basics of Arrays and Strings

Array

```cpp
int A[] = {3, 6, 8}; // A[] can be replaced with A[3]
std::cout << "A[0] = " << A[0] << std::endl; // prints 3
```

String as an array of characters

```cpp
char s[] = "Hello, world"; // or equivalently, char* s = "Hello, world"
std::cout << "s[0] = " << s[0] << std::endl; // prints 'H'
std::cout << "s = " << s << std::endl; // prints "Hello, world"
```
Assignment and Arithmetic Operators

```c
int a = 3, b = 2; // valid
int c = a + b; // addition: c == 5
int d = a - b; // subtraction: d == 1
int e = a * b; // multiplication: e == 6
int f = a / b; // division (int) generates quotient: f == 1
int g = a + b * c; // precedence - add after multiply: g == 3 + 2 * 5 == 13
a = a + 2; // a == 5
a += 2; // a == 7
++a; // a == 8
a = b = c = e; // a == b == c == e == 6
```
Comparison Operators and Conditional Statements

```cpp
int a = 2, b = 2, c = 3;
std::cout << (a == b) << std::endl; // prints 1 (true)
std::cout << (a == c) << std::endl; // prints 0 (false)
std::cout << (a != c) << std::endl; // prints 1 (true)
if ( a == b ) {
    std::cout << "a and b are same" << std::endl;
}
else {
    std::cout << "a and b are different" << std::endl;
}
std::cout << "a and b are " << (a == b ? "same" : "different") << std::endl
    << "a is " << (a < b ? "less" : "not less") << " than b" << std::endl
    << "a is " << (a <= b ? "equal or less" : "greater") << " than b" << std::endl;
```

Hyun Min Kang
Biostatistics 615/815 - Lecture 2
January 11th, 2011
Loops

while loop

```cpp
int i=0;  // initialize the key value
while( i < 10 ) {  // evaluate the loop condition
    std::cout << "i = " << i << std::endl;  // prints i=0 ... i=9
    ++i;  // update the key value
}
```

for loop

```cpp
for(int i=0; i < 10; ++i) {  // initialize, evaluate, update
    std::cout << "i = " << i << std::endl;  // prints i=0 ... i=9
}
```
Pointers

Another while loop

```cpp
char* s = "Hello"; // array of {'H','e','l','l','o','\0'}
while ( *s != '\0' ) { // *s access the character value pointed by s
    std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
    ++s; // advancing the pointer by one
}
```
Pointers and Loops

### while loop
```cpp
char* s = "Hello"; // array of {'H','e','l','l','o','\0'}
while ( *s != '\0' ) {
    std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
    ++s; // advancing the pointer by one
}
```

### for loop
```cpp
for(char* s = "Hello"; *s != '\0'; ++s) { //
    std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
}
```
Pointers are complicated, but important

```cpp
int A[] = {3, 6, 8}; // A is a pointer to a constant address
int* p = A;        // p and A are containing the same address
std::cout << p[0] << std::endl;    // prints 3 because p[0] == A[0] == 3
std::cout << *p << std::endl;      // prints 3 because *p == p[0]
std::cout << *(p + 2) << std::endl;   // prints 3 because *(p+2) == p[2]
int b = 3;   // regular integer value
int* q = &b; // the value of q is the address of b
b = 4;      // the value of b is changed
std::cout << *q << std::endl;     // *q == b == 4

char s[] = "Hello";
char *t = s;
std::cout << t << std::endl;      // prints "Hello"
char *u = &s[3]; // &s[3] is equivalent to s + 3
std::cout << u << std::endl;     // prints "lo"
```
Pointers and References

```cpp
int a = 2;
int& ra = a;   // reference to a
int* pa = &a;  // pointer to a
int b = a;     // copy of a
++a;           // increment a
std::cout << a << std::endl;  // prints 3
std::cout << ra << std::endl; // prints 3
std::cout << *pa << std::endl; // prints 3
std::cout << b << std::endl;  // prints 2
int* pb;       // valid, but what pb points to is undefined
int* pc = NULL; // valid, pc points to nothing
std::cout << *pc << std::endl; // Run-time error : pc cannot be dereferenced.
int& rb;       // invalid. reference must refer to something
int& rb = 2;   // invalid. reference must refer to a variable.
```
**Command line arguments**

```c
int main(int argc, char** argv)
```

**int argc**  
Number of command line arguments, including the program name itself

**char**\** argv**  
List of command line arguments as double pointer

- One * for representing 'array' of strings
- One * for representing string as 'array' of characters

- argv[0] represents the program name (e.g., **helloWorld**)
- argv[1] represents the first command-line argument
- argv[2] represents the second command-line argument
- ...  
- argv[argc-1] represents the last command-line argument
Handling command line arguments

echo.cpp - echoes command line arguments to the standard output

```cpp
#include <iostream>

int main(int argc, char** argv) {
    for(int i=1; i < argc; ++i) {
        if (i > 1) // print blank if there is an item already printed
            std::cout << " ";
        std::cout << argv[i]; // print each command line argument
    }
    std::cout << std::endl; // print end-of-line at the end
}
```

Compiling and running echo.cpp

```bash
user@host:~/$ g++ -o echo echo.cpp
user@host:~/$ ./echo 1 2 3 my name is foo
1 2 3 my name is foo
```
Functions

Core element of function

- **Type**: Type of return values
- **Arguments**: List of comma separated input arguments
- **Body**: Body of function with "return [value]" at the end

```c
int square(int a) {
    return (a*a);
}
```

```
int x = 5;
std::cout << square(x) << std::endl; // prints 25
```
### Functions

<table>
<thead>
<tr>
<th>Core element of function</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong> Type of return values</td>
<td></td>
</tr>
<tr>
<td><strong>Arguments</strong> List of comma separated input arguments</td>
<td></td>
</tr>
<tr>
<td><strong>Body</strong> Body of function with &quot;return [value]&quot; at the end</td>
<td></td>
</tr>
</tbody>
</table>

### Defining functions

```cpp
int square(int a) {
    return (a*a);
}
```
Functions

Core element of function

- **Type**: Type of return values
- **Arguments**: List of comma separated input arguments
- **Body**: Body of function with "return [value]" at the end

Defining functions

```cpp
int square(int a) {
    return (a*a);
}
```

Calling functions

```cpp
int x = 5;
std::cout << square(x) << std::endl; // prints 25
```
Call by value vs. Call by reference

callByValRef.cpp

```cpp
#include <iostream>

int foo(int a) {
    a = a + 1;
    return a;
}

int bar(int& a) {
    a = a + 1;
    return a;
}

int main(int argc, char** argv) {
    int x = 1, y = 1;
    std::cout << foo(x) << std::endl; // prints 2
    std::cout << x << std::endl; // prints 1
    std::cout << bar(y) << std::endl; // prints 2
    std::cout << y << std::endl; // prints 2
}
```
Let's implement Fisher's exact Test

A 2 × 2 table

<table>
<thead>
<tr>
<th></th>
<th>Placebo</th>
<th>Treatment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diseased</td>
<td>a</td>
<td>b</td>
<td>a+b</td>
</tr>
<tr>
<td>Cured</td>
<td>c</td>
<td>d</td>
<td>c+d</td>
</tr>
<tr>
<td>Total</td>
<td>a+c</td>
<td>b+d</td>
<td>n</td>
</tr>
</tbody>
</table>
Let’s implement Fisher’s exact Test

A $2 \times 2$ table

<table>
<thead>
<tr>
<th></th>
<th>Placebo</th>
<th>Treatment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diseased</td>
<td>a</td>
<td>b</td>
<td>a+b</td>
</tr>
<tr>
<td>Cured</td>
<td>c</td>
<td>d</td>
<td>c+d</td>
</tr>
<tr>
<td>Total</td>
<td>a+c</td>
<td>b+d</td>
<td>n</td>
</tr>
</tbody>
</table>

Desired Program Interface and Results

```
user@host:~/$ ./fishersExactTest 1 2 3 0
Two-sided p-value is 0.4

user@host:~/$ ./fishersExactTest 2 7 8 2
Two-sided p-value is 0.0230141

user@host:~/$ ./fishersExactTest 20 70 80 20
Two-sided p-value is 5.90393e-16
```
Fisher’s Exact Test

Possible 2 × 2 tables

<table>
<thead>
<tr>
<th></th>
<th>Placebo</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disease</td>
<td>x</td>
<td>a+b-x</td>
</tr>
<tr>
<td>Cured</td>
<td>a+c-x</td>
<td>d-a+x</td>
</tr>
<tr>
<td>Total</td>
<td>a+c</td>
<td>b+d</td>
</tr>
</tbody>
</table>

Hypergeometric distribution

Given \( a + b, c + d, a + c, b + d \) and \( n = a + b + c + d \),

\[
Pr(x) = \frac{(a + b)!(c + d)!(a + c)!(b + d)!}{x!(a + b - x)!(a + c - x)!(d - a + x)!n!}
\]

Fishers’s Exact Test (2-sided)

\[
p_{FET}(a, b, c, d) = \sum_x Pr(x)I[Pr(x) \leq Pr(a)]
\]
#include <iostream>

double hypergeometricProb(int a, int b, int c, int d); // defined later

int main(int argc, char** argv) {
    // read input arguments
    int a = atoi(argv[1]), b = atoi(argv[2]), c = atoi(argv[3]), d = atoi(argv[4]);
    int n = a + b + c + d;
    // find cutoff probability
    double pCutoff = hypergeometricProb(a, b, c, d);
    double pValue = 0;
    // sum over probability smaller than the cutoff
    for(int x=0; x <= n; ++x) { // among all possible x
        if ( a+b-x >= 0 && a+c-x >= 0 && d-a+x >=0 ) { // consider valid x
            double p = hypergeometricProb(x, a+b-x, a+c-x, d-a+x);
            if ( p <= pCutoff ) pValue += p;
        }
    }
    std::cout << "Two-sided p-value is " << pValue << std::endl;
    return 0;
}
```cpp
int fac(int n) { // calculates factorial
    int ret;
    for(ret=1; n > 0; --n) { ret *= n; }
    return ret;
}

double hypergeometricProb(int a, int b, int c, int d) {
    int num = fac(a+b) * fac(c+d) * fac(a+c) * fac(b+d);
    int den = fac(a) * fac(b) * fac(c) * fac(d) * fac(a+b+c+d);
    return (double)num/(double)den;
}
```

Running Examples

```
user@host:~/$ ./intFishersExactTest 1 2 3 0
Two-sided p-value is 0.4     // correct
user@host:~/$ ./intFishersExactTest 2 7 8 2
Two-sided p-value is 4.41018 // INCORRECT
```
Considering Precision Carefully

```cpp
int fac(int n) { // calculates factorial
    int ret;
    for(ret=1; n > 0; --n) { ret *= n; }
    return ret;
}

int main(int argc, char** argv) {
    int n = atoi(argv[1]);
    std::cout << n << "! = " << fac(n) << std::endl;
}
```

Running Examples

```
user@host:~/$ ./factorial 10
10! = 362880 // correct
user@host:~/$ ./factorial 12
12! = 479001600 // correct
user@host:~/$ ./factorial 13
13! = 1932053504 // INCORRECT
```
doubleFishersExactTest.cpp

new hypergeometricProb() function

double fac(int n) { // main() function remains the same
    double ret; // use double instead of int
    for(ret=1.; n > 0; --n) { ret *= n; }
    return ret;
}
double hypergeometricProb(int a, int b, int c, int d) {
    double num = fac(a+b) * fac(c+d) * fac(a+c) * fac(b+d);
    double den = fac(a) * fac(b) * fac(c) * fac(d) * fac(a+b+c+d);
    return num/den; // use double to calculate factorials
}

Running Examples

user@host:~/$ ./doubleFishersExactTest 2 7 8 2
Two-sided p-value is 0.023041
user@host:~/$ ./doubleFishersExactTest 20 70 80 20
Two-sided p-value is 0 (fac(190) > 1e308 - beyond double precision)
How to perform Fisher’s exact test with large values

Problem - Limited Precision

- int handles only up to fac(12)
- double handles only up to fac(170)

Solution - Calculate in logarithmic scale

\[
\log \Pr(x) = \log((a + b)! + \log(c + d)! + \log(a + c)! + \log(b + d)! - \log x! \\
- \log(a + b - x)! - \log(a + c - x)! - \log(d - a + x)! - \log n!)
\]

\[
\log(p_{FET}) = \log \left[ \sum_x \Pr(x)I(\Pr(x) \leq \Pr(a)) \right]
\]

\[
= \log \Pr(a) + \log \left[ \sum_x \exp(\log \Pr(x) - \log \Pr(a)) I(\log \Pr(x) \leq \log \Pr(a)) \right]
\]
logFishersExactTest.cpp - main() function

```cpp
#include <iostream>
#include <math> // for calculating log() and exp()

double logHypergeometricProb(int a, int b, int c, int d); // defined later

int main(int argc, char** argv) {
    int a = atoi(argv[1]), b = atoi(argv[2]), c = atoi(argv[3]), d = atoi(argv[4]);
    int n = a + b + c + d;
    double logpCutoff = logHypergeometricProb(a, b, c, d);
    double pFraction = 0;
    for(int x=0; x <= n; ++x) { // among all possible x
        if ( a+b-x >= 0 && a+c-x >= 0 && d-a+x >=0 ) { // consider valid x
            double l = logHypergeometricProb(x, a+b-x, a+c-x, d-a+x);
            if ( l <= logpCutoff ) pFraction += exp(l - logpCutoff);
        }
    }
    double logpValue = logpCutoff + log(pFraction);
    std::cout << "Two-sided log10-p-value is " << logpValue/log(10.) << std::endl;
    std::cout << "Two-sided p-value is " << exp(logpValue) << std::endl;
    return 0;
}
```
Filling the rest

**logHypergeometricProb()**

```c
double logFac(int n) {
    double ret;
    for(ret=0.; n > 0; --n) { ret += log((double)n); } 
    return ret;
}

double logHypergeometricProb(int a, int b, int c, int d) {
    return logFac(a+b) + logFac(c+d) + logFac(a+c) + logFac(b+d) - logFac(a)
    - logFac(b) - logFac(c) - logFac(d) - logFac(a+b+c+d);
}
```

**Running Examples**

```bash
user@host:~/$ ./logFishersExactTest 2 7 8 2
Two-sided log10-p-value is -1.63801, p-value is 0.0230141
user@host:~/$ ./logFishersExactTest 20 70 80 20
Two-sided log10-p-value is -15.2289, p-value is 5.90393e-16
user@host:~/$ ./logFishersExactTest 200 700 800 200
Two-sided log10-p-value is -147.563, p-value is 2.73559e-148
```
Even faster

Computational speed for large dataset

```bash
time ./logFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m42.614s

time ./fastLogFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m0.007s
```
Even faster

Computational speed for large dataset

```
time ./logFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m42.614s

time ./fastLogFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m0.007s
```

How to make it faster?

*To be continued...*
Summary so far

- Algorithms are computational steps
- `towerOfHanoi` utilizing recursions
- `insertionSort`
  - ✓ Simple but a slow sorting algorithm.
  - ✓ Loop invariant property
- Data types and floating-point precisions
- Operators, `if`, `for`, and `while` statements
- Arrays and strings
- Pointers and References
- Functions
- Fisher’s Exact Test
  - ✓ `intFishersExactTest` - works only tiny datasets
  - ✓ `doubleFishersExactTest` - handles small datasets
  - ✓ `logFishersExactTest` - handles hundreds of observations
- At Home: Reading material for novice C++ users: 
At Home: Write, Compile and Run.

The following list of programs:

- helloWorld.cpp
- towerOfHanoi.cpp
- insertionSort.cpp
- echo.cpp
- precisionExample.cpp
- callByValRef.cpp
- factorial.cpp
- intFishersExactTest.cpp
- doubleFishersExactTest.cpp
- logFishersExactTest.cpp
### At Home: Write, Compile and Run...

#### The following list of programs

<table>
<thead>
<tr>
<th>Program Name</th>
<th>Program Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>helloWorld.cpp</td>
<td>callByValRef.cpp</td>
</tr>
<tr>
<td>towerOfHanoi.cpp</td>
<td>factorial.cpp</td>
</tr>
<tr>
<td>insertionSort.cpp</td>
<td>intFishersExactTest.cpp</td>
</tr>
<tr>
<td>echo.cpp</td>
<td>doubleFishersExactTest.cpp</td>
</tr>
<tr>
<td>precisionExample.cpp</td>
<td>logFishersExactTest.cpp</td>
</tr>
</tbody>
</table>

#### How to...

<table>
<thead>
<tr>
<th>Task</th>
<th>Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write</td>
<td>Notepad, Vim, Emacs, Eclipse, VisualStudio, etc</td>
</tr>
<tr>
<td>Compile</td>
<td>g++ -Wall -o [progName] [progName].cpp</td>
</tr>
<tr>
<td></td>
<td>(Unix, Mac OS X, or Cygwin)</td>
</tr>
<tr>
<td>Run</td>
<td>./[progName] [list of arguments]</td>
</tr>
</tbody>
</table>
Next Lecture

Fisher’s Exact Test
- fastLogFishersExactTest
- oneSidedFastLogFishersExactTest - First homework

More on C++ Programming
- Standard Template Library
- User-defined data types

Divide and Conquer Algorithms
- Binary Search
- Merge Sort