Biostatistics 615/815
Implementing Algorithms in C++

Hyun Min Kang

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Recap from the Last Lecture

- Algorithm : A sequence of computational steps from input to ouput
  - ✓ SingOldMacDonaldSong
  - ✓ InsertionSort
  - ✓ TowerOfHanoi
- Implemetation in C++
  - ✓ `helloWorld`
  - ✓ `towerOfHanoi`
  - ✓ `insertionSort` (skipped)

Recap - helloWorld

Writing helloWorld.cpp

```
#include <iostream> // import input/output handling library
int main(int argc, char** argv) {
  std::cout << "Hello, World" << std::endl;
  return 0; // program exits normally
}
```

Compiling helloWorld.cpp

Install Cygwin (Windows), Xcode (MacOS), or nothing (Linux).

```
user@host:~/$ g++ -o helloWorld helloWorld.cpp
```

Running helloWorld

```
user@host:~/$ ./helloWorld
Hello, World
```

How helloWorld works

main() : First function to be called

```
// type of return value is integer
// return value of main() function is program exit code
// 0 is normal exit code and the others are abnormal exit codes
int
// name (identifier) of function is 'main'
// 'main' is a special function, invoked at the beginning of a program.
main
(                // function arguments are surrounded by parentheses
  int argc,      // number of command line arguments
  char** argv    // list of command line arguments - will explain later
)
{
  // ... function body goes here
  return 0; // return normal exit code
}
```

How helloWorld works

Using iostream to output strings to console

```
// includes standard library for handling I/Os (inputs/outputs)
// std::cout and std::endl cannot be recognized without including <iostream>
#include <iostream>
int main (int argc, char** argv) {
  std::cout // standard output stream - messages are printed to console.
    << // insertion operator : appends the next value to the output stream
    "Hello, World" // string appended to std::cout via operator<<
    << // insertion operator : appends the next value to the output stream
    std::endl; // end-of-line appended to std::cout via operator<<
  return 0;
}
```

Today's Lecture

What to expect

- Basic Data Types
- Control Structures
- Pointers and Functions

Today's Lecture

What to expect

- Basic Data Types
- Control Structures
- Pointers and Functions

What are expected for the next few weeks

- The class does NOT focus on teaching programming language itself
- Expect to spend time to be familar to programming languages
  - ✓ Online reference : *http://www.cplusplus.com/doc/tutorial/*
  - ✓ Offline reference : C++ Primer Plus, 5th Edition
- VERY important to practice writing code on your own.
- Utilize office hours or after-class minutes for detailed questions in practice

Declaring Variables

Variable Declaration and Assignment

```
int foo; // declare a variable
foo = 5; // assign a value to a variable.
int foo = 5; // declararion + assignment
```

Declaring Variables

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int foo = 5; // declararion + assignment
```

Variable Names

```
int poodle; // valid
int Poodle; // valid and distinct from poodle
int my_stars3; // valid to include underscores and digits
int 4ever;  // invalid because it starts with a digit
int double; // invalid because double is C++ keyword
int honky-tonk; // invalid -- no hyphens allowed
```

Data Types

Signed Integer Types

```
short foo; // 16 bits (2 bytes) : -32,768 <= foo <= 32,767
int foo;   // Mostly 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long foo;  // 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long long foo; // 64 bits
short foo = 0;
foo = foo - 1;    // foo is -1
```

Data Types

Signed Integer Types

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short foo; // 16 bits (2 bytes) : -32,768 <= foo <= 32,767
int foo;   // Mostly 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long foo;  // 32 bits (4 bytes) : -2,147,483,648 <= foo <= 2,147,483,647
long long foo; // 64 bits
short foo = 0;
foo = foo - 1;    // foo is -1
```

Unsigned Integer Types

```
unsigned short foo; // 16 bits (2 bytes) : 0 <= foo <= 65,535
unsigned int foo;   // Mostly 32 bits (4 bytes) : 0 <= foo <= 4,294,967,295
unsigned long foo;  // 32 bits (4 bytes) : 0 <= foo <= 4,294,967,295
unsigned long long foo; // 64 bits
unsigned short foo = 0;
foo = foo - 1;    // foo is 65,535
```

Floating Point Numbers

Comparisons

Type	`float`	`double`	`long double`
Precision	Single	Double	Quadruple
Size	32 bits	64 bits	128 bits
(in most modern OS)	4 bytes	8 bytes	16 bytes
Sign	1 bit	1 bit	1 bit
Exponent	8 bits	11 bits	15 bits
Fraction	23 bits	52 bits	112 bits
(# decimal digits)	7.2	16	34
Minimum (>0)	1.2×10^{-38}	2.2×10^{-308}	3.4×10^{-4932}
Maximum	3.4×10^{38}	1.8×10^{308}	1.2×10^{4932}

Handling Floating Point Precision Carefully

precisionExample.cpp

```
#include <iostream>
int main(int argc, char** argv) {
  float smallFloat = 1e-8; // a small value
  float largeFloat = 1.;   // difference in 8 (>7.2) decimal figures.
  std::cout << smallFloat << std::endl; // "1e-08" is printed
  smallFloat = smallFloat + largeFloat; // smallFloat becomes exactly 1
  smallFloat = smallFloat - largeFloat; // smallFloat becomes exactly 0
  std::cout << smallFloat << std::endl; // "0" is printed
  // similar thing happens for doubles (e.g. 1e-20 vs 1).
  return 0;
}
```

Basics of Arrays and Strings

Array

```
int A[] = {3,6,8}; // A[] can be replaced with A[3]
std::cout << "A[0] = " << A[0] << std::endl; // prints 3
std::cout << "A[1] = " << A[1] << std::endl; // prints 6
std::cout << "A[2] = " << A[2] << std::endl; // prints 8
```

String as an array of characters

```
char s[] = "Hello, world"; // or equivalently, char* s = "Hello, world"
std::cout << "s[0] = " << s[0] << std::endl; // prints 'H'
std::cout << "s[5] = " << s[5] << std::endl; // prints ','
std::cout << "s = " << s << std::endl; // prints "Hello, world"
```

Assignment and Arithmetic Operators

```
int a = 3, b = 2;  // valid
int c = a + b;     // addition : c == 5
int d = a - b;     // subtraction : d == 1
int e = a * b;     // multiplication : e == 6
int f = a / b;     // division (int) generates quotient : f == 1
int g = a + b * c; // precedence - add after multiply : g == 3 + 2 * 5 == 13
a = a + 2;         // a == 5
a += 2;            // a == 7
++a;               // a == 8
a = b = c = e;     // a == b == c == e == 6
```

Comparison Operators and Conditional Statements

```
int a = 2, b = 2, c = 3;
std::cout << (a == b) << std::endl; // prints 1 (true)
std::cout << (a == c) << std::endl; // prints 0 (false)
std::cout << (a != c) << std::endl; // prints 1 (true)
if ( a == b ) {  // conditional statment
  std::cout << "a and b are same" << std::endl;
}
else {
  std::cout << "a and b are different" << std::endl;
}
std::cout << "a and b are " << (a == b ? "same" : "different") << std::endl
  << "a is " << (a < b ? "less" : "not less") << " than b" << std::endl
  << "a is " << (a <= b ? "equal or less" : "greater") << " than b" << std::endl;
```

Loops

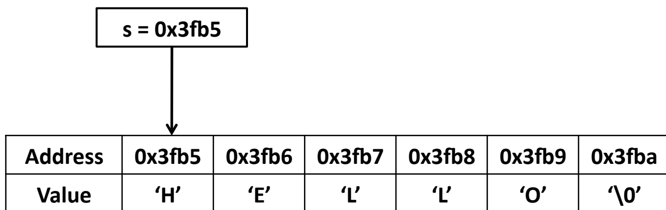
while loop

```
int i=0;  // initialize the key value
while( i < 10 ) { // evaluate the loop condition
  std::cout << "i = " << i << std::endl; // prints i=0 ... i=9
  ++i; // update the key value
}
```

for loop

```
for(int i=0; i < 10; ++i) { // initialize, evaluate, update
  std::cout << "i = " << i << std::endl; // prints i=0 ... i=9
}
```

Pointers



Another while loop

```
char* s = "Hello"; // array of {'H','e','l','l','o','\0'}
while ( *s != '\0' ) {  // *s access the character value pointed by s
  std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
  ++s; // advancing the pointer by one
}
```

Pointers and Loops

while loop

```
char* s = "Hello"; // array of {'H','e','l','l','o','\0'}
while ( *s != '\0' ) {
  std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
  ++s; // advancing the pointer by one
}
```

for loop

```
for(char* s = "Hello"; *s != '\0'; ++s) { //
  std::cout << *s << std::endl; // prints 'H','e','l','l','o' at each line
}
```

Pointers are complicated, but important

```
int A[] = {3,6,8}; // A is a pointer to a constant address
int* p = A;        // p and A are containing the same address
std::cout << p[0] << std::endl;   // prints 3 because p[0] == A[0] == 3
std::cout << *p << std::endl;   // prints 3 because *p == p[0]
std::cout << p[2] << std::endl; // prints 8 because p[2] == A[2] == 8
std::cout << *(p+2) << std::endl;   // prints 3 because *(p+2) == p[2]
int b = 3;     // regular integer value
int* q = &b;   // the value of q is the address of b
b = 4;         // the value of b is changed
std::cout << *q << std::endl;  // *q == b == 4

char s[] = "Hello";
char *t = s;
std::cout << t << std::endl; // prints "Hello"
char *u = &s[3]; // &s[3] is equivalent to s + 3
std::cout << u << std::endl; // prints "lo"
```

Pointers and References

```
int a = 2;
int& ra = a;   // reference to a
int* pa = &a;  // pointer to a
int b = a;     // copy of a
++a;  // increment a
std::cout << a << std::endl;  // prints 3
std::cout << ra << std::endl; // prints 3
std::cout << *pa << std::endl; // prints 3
std::cout << b << std::endl; // prints 2
int* pb;        // valid, but what pb points to is undefined
int* pc = NULL; // valid, pc points to nothing
std::cout << *pc << std::endl; // Run-time error : pc cannot be dereferenced.
int& rb;        // invalid. reference must refer to something
int& rb = 2;    // invalid. reference must refer to a variable.
```

Command line arguments

`int main(int argc, char** argv)`

`int argc` Number of command line arguments, including the program name itself

`char** argv` List of command line arguments as double pointer

- One * for representing 'array' of strings
- One * for representing string as 'array' of characters

✓ `argv[0]` represents the program name (e.g., `helloWorld`)
✓ `argv[1]` represents the first command-line argument
✓ `argv[2]` represents the second command-line argument
✓ ...
✓ `argv[argc-1]` represents the last command-line argument

Handling command line arguments

echo.cpp - echoes command line arguments to the standard output

```
#include <iostream>
int main(int argc, char** argv) {
  for(int i=1; i < argc; ++i) { // i=1 : 2nd argument (skip program name)
    if ( i > 1 ) // print blank if there is an item already printed
      std::cout << " ";
    std::cout << argv[i];  // print each command line argument
  }
  std::cout << std::endl; // print end-of-line at the end
}
```

Compiling and running echo.cpp

```
user@host:~/$ g++ -o echo echo.cpp
user@host:~/$ ./echo 1 2 3 my name is foo
1 2 3 my name is foo
```

Functions

Core element of function

- **Type** Type of return values
- **Arguments** List of comma separated input arguments
- **Body** Body of function with "return [value]" at the end

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Defining functions

```
int square(int a) {
  return (a*a);
}
```

Functions

Core element of function

Type	Type of return values
Arguments	List of comma separated input arguments
Body	Body of function with "return [value]" at the end

Defining functions

```
int square(int a) {
  return (a*a);
}
```

Calling functions

```
int x = 5;
std::cout << square(x) << std::endl; // prints 25
```

Call by value vs. Call by reference

callByValRef.cpp

```
#include <iostream>
int foo(int a) {
  a = a + 1;
  return a;
}
int bar(int& a) {
  a = a + 1;
  return a;
}
int main(int argc, char** argv) {
  int x = 1, y = 1;
  std::cout << foo(x) << std::endl; // prints 2
  std::cout << x << std::endl;      // prints 1
  std::cout << bar(y) << std::endl; // prints 2
  std::cout << y << std::endl;      // prints 2
}
```

Let's implement Fisher's exact Test

A 2×2 table

	Placebo	Treatment	Total
Diseased	a	b	a+b
Cured	c	d	c+d
Total	a+c	b+d	n

Let's implement Fisher's exact Test

A 2×2 table

	Placebo	Treatment	Total
Diseased	a	b	a+b
Cured	c	d	c+d
Total	a+c	b+d	n

Desired Program Interface and Results

```
user@host:~/$ ./fishersExactTest 1 2 3 0
Two-sided p-value is 0.4
user@host:~/$ ./fishersExactTest 2 7 8 2
Two-sided p-value is 0.0230141
user@host:~/$ ./fishersExactTest 20 70 80 20
Two-sided p-value is 5.90393e-16
```

Fisher's Exact Test

Possible 2×2 tables

	Placebo	Treatment	Total
Diseased	x	a+b-x	a+b
Cured	a+c-x	d-a+x	c+d
Total	a+c	b+d	n

Hypergeometric distribution

Given $a+b, c+d, a+c, b+d$ and $n = a+b+c+d$,

$$\Pr(x) = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{x!(a+b-x)!(a+c-x)!(d-a+x)!n!}$$

Fishers's Exact Test (2-sided)

$p_{FET}(a,b,c,d) = \sum_x \Pr(x) I[\Pr(x) \leq \Pr(a)]$

intFishersExactTest.cpp - main() function

```
#include <iostream>
double hypergeometricProb(int a, int b, int c, int d); // defined later
int main(int argc, char** argv) {
  // read input arguments
  int a = atoi(argv[1]), b = atoi(argv[2]), c = atoi(argv[3]), d = atoi(argv[4]);
  int n = a + b + c + d;
  // find cutoff probability
  double pCutoff = hypergeometricProb(a,b,c,d);
  double pValue = 0;
  // sum over probability smaller than the cutoff
  for(int x=0; x <= n; ++x) { // among all possible x
    if ( a+b-x >= 0 && a+c-x >= 0 && d-a+x >=0 ) { // consider valid x
      double p = hypergeometricProb(x,a+b-x,a+c-x,d-a+x);
      if ( p <= pCutoff ) pValue += p;
    }
  }
  std::cout << "Two-sided p-value is " << pValue << std::endl;
  return 0;
}
```

intFishersExactTest.cpp

hypergeometricProb() function

```
int fac(int n) { // calculates factorial
  int ret;
  for(ret=1; n > 0; --n) { ret *= n; }
  return ret;
}
double hypergeometricProb(int a, int b, int c, int d) {
  int num = fac(a+b) * fac(c+d) * fac(a+c) * fac(b+d);
  int den = fac(a) * fac(b) * fac(c) * fac(d) * fac(a+b+c+d);
  return (double)num/(double)den;
}
```

Running Examples

```
user@host:~/$ ./intFishersExactTest 1 2 3 0
Two-sided p-value is 0.4      // correct
user@host:~/$ ./intFishersExactTest 2 7 8 2
Two-sided p-value is 4.41018  // INCORRECT
```

Considering Precision Carefully

factorial.cpp

```
int fac(int n) { // calculates factorial
  int ret;
  for(ret=1; n > 0; --n) { ret *= n; }
  return ret;
}
int main(int argc, char** argv) {
  int n = atoi(argv[1]);
  std::cout << n << "! = " << fac(n) << std::endl;
}
```

Running Examples

```
user@host:~/$ ./factorial 10
10! = 362880      // correct
user@host:~/$ ./factorial 12
12! = 479001600  // correct
user@host:~/$ ./factorial 13
13! = 1932053504 // INCORRECT
```

doubleFishersExactTest.cpp

new hypergeometricProb() function

```
double fac(int n) { // main() function remains the same
  double ret; // use double instead of int
  for(ret=1.; n > 0; --n) { ret *= n; }
  return ret;
}
double hypergeometricProb(int a, int b, int c, int d) {
  double num = fac(a+b) * fac(c+d) * fac(a+c) * fac(b+d);
  double den = fac(a) * fac(b) * fac(c) * fac(d) * fac(a+b+c+d);
  return num/den; // use double to calculate factorials
}
```

Running Examples

```
user@host:~/$ ./doubleFishersExactTest 2 7 8 2
Two-sided p-value is 0.023041
user@host:~/$ ./doubleFishersExactTest 20 70 80 20
Two-sided p-value is 0 (fac(190) > 1e308 - beyond double precision)
```

How to perform Fisher's exact test with large values

Problem - Limited Precision

- `int` handles only up to fac(12)
- `double` handles only up to fac(170)

Solution - Calculate in logarithmic scale

$$
\begin{aligned}
\log \Pr(x) &= \log(a+b)! + \log(c+d)! + \log(a+c)! + \log(b+d)! - \log x! \\
&\quad - \log(a+b-x)! - \log(a+c-x)! - \log(d-a+x)! - \log n! \\
\log(p_{FET}) &= \log \left[\sum_x \Pr(x) I(\Pr(x) \leq \Pr(a)) \right] \\
&= \log \Pr(a) + \log \left[\sum_x \exp(\log \Pr(x) - \log \Pr(a)) I(\log \Pr(x) \leq \log \Pr(a)) \right]
\end{aligned}
$$

logFishersExactTest.cpp - main() function

```
#include <iostream>
#include <math> // for calculating log() and exp()
double logHypergeometricProb(int a, int b, int c, int d); // defined later
int main(int argc, char** argv) {
  int a = atoi(argv[1]), b = atoi(argv[2]), c = atoi(argv[3]), d = atoi(argv[4]);
  int n = a + b + c + d;
  double logpCutoff = logHypergeometricProb(a,b,c,d);
  double pFraction = 0;
  for(int x=0; x <= n; ++x) { // among all possible x
    if ( a+b-x >= 0 && a+c-x >= 0 && d-a+x >=0 ) { // consider valid x
      double l = logHypergeometricProb(x,a+b-x,a+c-x,d-a+x);
      if ( l <= logpCutoff ) pFraction += exp(l - logpCutoff);
    }
  }
  double logpValue = logpCutoff + log(pFraction);
  std::cout << "Two-sided log10-p-value is " << logpValue/log(10.) << std::endl;
  std::cout << "Two-sided p-value is " << exp(logpValue) << std::endl;
  return 0;
}
```

Filling the rest

logHypergeometricProb()

```
double logFac(int n) {
  double ret;
  for(ret=0.; n > 0; --n) { ret += log((double)n); }
  return ret;
}
double logHypergeometricProb(int a, int b, int c, int d) {
  return logFac(a+b) + logFac(c+d) + logFac(a+c) + logFac(b+d) - logFac(a)
      - logFac(b) - logFac(c) - logFac(d) - logFac(a+b+c+d);
}
```

Running Examples

```
user@host:~/$ ./logFishersExactTest 2 7 8 2
Two-sided log10-p-value is -1.63801, p-value is 0.0230141
user@host:~/$ ./logFishersExactTest 20 70 80 20
Two-sided log10-p-value is -15.2289, p-value is 5.90393e-16
user@host:~/$ ./logFishersExactTest 200 700 800 200
Two-sided log10-p-value is -147.563, p-value is 2.73559e-148
```

Even faster

Computational speed for large dataset

```
time ./logFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m42.614s

time ./fastLogFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m0.007s
```

Even faster

Computational speed for large dataset

```
time ./logFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m42.614s

time ./fastLogFishersExactTest 2000 7000 8000 2000
Two-sided log10-p-value is -1466.13, p-value is 0
real 0m0.007s
```

How to make it faster?

To be continued...

Summary so far

- Algorithms are computational steps
- `towerOfHanoi` utilizing recursions
- `insertionSort`
  - ✓ Simple but a slow sorting algorithm.
  - ✓ Loop invariant property
- Data types and floating-point precisions
- Operators, `if`, `for`, and `while` statements
- Arrays and strings
- Pointers and References
- Functions
- Fisher's Exact Test
  - ✓ `intFishersExactTest` - works only tiny datasets
  - ✓ `doubleFishersExactTest` - handles small datasets
  - ✓ `logFishersExactTest` - handles hundreds of observations
- At Home : Reading material for novice C++ users : *http://www.cplusplus.com/doc/tutorial/*

At Home : Write, Compile and Run..

The following list of programs

- `helloWorld.cpp`
- `towerOfHanoi.cpp`
- `insertionSort.cpp`
- `echo.cpp`
- `precisionExample.cpp`
- `callByValRef.cpp`
- `factorial.cpp`
- `intFishersExactTest.cpp`
- `doubleFishersExactTest.cpp`
- `logFishersExactTest.cpp`

At Home : Write, Compile and Run..

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- `doubleFishersExactTest.cpp`
- `logFishersExactTest.cpp`

How to ...

Write Notepad, Vim, Emacs, Eclipse, VisualStudio, etc

Compile `g++ -Wall -o [progName] [progName].cpp`
(Unix, Mac OS X, or Cygwin)

Run `./[progName] [list of arguments]`

Next Lecture

Fisher's Exact Test

- `fastLogFishersExactTest`
- `oneSidedFastLogFishersExactTest` - First homework

More on C++ Programming

- Standard Template Library
- User-defined data types

Divide and Conquer Algorithms

- Binary Search
- Merge Sort