

Biostatistics 615/815 Lecture 5: More on STLs, and Divide and Conquer Algorithms

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Announcements

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- When classroom is full, the seating priority should be given to enrolled students

Recap on Classes

```

#include <iostream>
class Point {
public:
    int x, y;
    Point(int px, int py) : x(px), y(py) { std::cout << "Point::Point(int,int)" << std::endl;}
    Point() : x(0), y(0) { std::cout << "Point::Point()" << std::endl; }
    void print() { std::cout << "(" << x << ", " << y << ")" << std::endl; }
};

int main(int argc, char** argv) { // what happens at each line?
    Point p1; // Point::Point()
    Point p2(3,4); // Point::Point(int,int)
    Point* pp1 = new Point(5,12); // Point::Point(int,int)
    Point* pp2 = new Point[5]; // Point::Point() (5 times)
    p1.print(); // (0,0)
    p2.print(); // (3,4)
    pp1->print(); // (3,4)
    pp2->print(); // (0,0)
    pp2[5].print(); // (0,0)
    delete pp1;
    delete [] pp2;
    return 0;
}

```

Recap on STL : vector

```
#include <iostream>
#include <vector>
int main(int argc, char** argv) {
    // vector examples
    std::vector<int> v;
    v.push_back(10);           // v contains {10}
    v.push_back(5);           // v contains {10,5}
    v.insert(v.begin(),7);    // v contains {7,10,5}
    v[2] = 1;                 // v contains {7,10,1};
    std::sort(v.begin(),v.end()); // v contains {1,7,10};
    std::cout << v[1] << std::endl; // prints 7
    return 0;
}
```

Recap on STL : string and map

```

#include <iostream>
#include <string>
#include <map>
int main(int argc, char** argv) {
    // string examples
    std::string s("hell");
    std::cout << s << std::endl;    // prints "hell"
    s += "o";                       // use '+' for concatenation
    std::cout << s << std::endl;    // prints "hello"
    s[0] = 'j';                     // access & assign individual character via []
    std::cout << s << std::endl;    // prints "jello"
    std::sort(s.begin(), s.end()); // string is a vector of characters
    std::cout << s << std::endl;    // prints "ejllo";
    // map examples
    std::map<std::string,int> points;
    points["Carlo Sidore"] = 100;
    points["Goncalo Abecasis"] = 99;
    points["Hyun Min Kang"] = 50;
    std::cout << points["Carlo Sidore"] << std::endl; // prints 100
    std::cout << points["Goncalo Abecasis"] << std::endl; // prints 99
    std::cout << points["Hyun Min Kim"] << std::endl; // nonexistent key; prints 0
}

```

Reading from Files : stdSort.cpp

```

#include <iostream>
#include <fstream>
#include <vector>
int main(int argc, char** argv) { // sorting software using std::sort
    int tok;
    std::vector<int> v;
    if ( argc > 1 ) { // if argument is given, read from file
        std::ifstream fin(argv[1]);
        while( fin >> tok ) { v.push_back(tok); }
        fin.close();
    }
    else { // read from standard input if no argument is specified
        while( std::cin >> tok ) { v.push_back(tok); }
    }
    std::sort(v.begin(), v.end()); // Sort using the algorithm in STL
    for(int i=0; i < v.size(); ++i) {
        std::cout << v[i] << std::endl; // print out the content
    }
    return 0;
}

```


Reading from Files : insertionSort.cpp

```
#include <iostream>
#include <fstream>
#include <vector>
void insertionSort(std::vector<int>& v); // insertionSort as defined before
int main(int argc, char** argv) {
    int tok;
    std::vector<int> v;
    if ( argc > 1 ) {
        std::ifstream fin(argv[1]);
        while( fin >> tok ) { v.push_back(tok); }
        fin.close();
    }
    else {
        while( std::cin >> tok ) { v.push_back(tok); }
    }
    insertionSort(v); // differs from stdSort in only this part
    for(int i=0; i < v.size(); ++i) {
        std::cout << v[i] << std::endl;
    }
    return 0;
}
```

Running time comparison

Running example with 100,000 elements (in UNIX or MacOS)

```
user@host:~/> time cat src/sample.input.txt | src/stdSort > /dev/null
real 0m0.430s
user 0m0.281s
sys 0m0.130s
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user@host:~/> time cat src/sample.input.txt | src/insertionSort > /dev/null
real 1m8.795s
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Recursion

Defintion of recursion

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Key components of recursion

- A function that is part of its own definition
- Terminating condition (to avoid infinite recursion)

Example of recursion

Factorial

```
int factorial(int n) {  
    if ( n == 0 )  
        return 1;  
    else  
        return n * factorial(n-1); // tail recursion - can be transformed into loop  
}
```

Example of recursion

Factorial

```
int factorial(int n) {
    if ( n == 0 )
        return 1;
    else
        return n * factorial(n-1); // tail recursion - can be transformed into loop
}
```

towerOfHanoi

```
void towerOfHanoi(int n, int s, int i, int d) { // n disks, from s to d via i
    if ( n > 0 ) {
        towerOfHanoi(n-1,s,d,i); // recursively move n-1 disks from s to i
        // Move n-th disk from s to d
        std::cout << "Disk " << n << " : " << s << " -> " << d << std::endl;
        towerOfHanoi(n-1,i,s,d); // recursively move n-1 disks from i to d
    }
}
```


Euclid's algorithm

Algorithm GCD

Data: Two integers a and b

Result: The greatest common divisor (GCD) between a and b

if a divides b **then**

 | **return** a

else

 | Find the largest integer t such that $at + r = b$;

 | **return** $\text{GCD}(r, a)$

end

Euclid's algorithm

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Result: The greatest common divisor (GCD) between a and b

if a divides b **then**

return a

else

 Find the largest integer t such that $at + r = b$;

return $\text{GCD}(r, a)$

end

Function gcd()

```
int gcd (int a, int b) {  
    if ( a == 0 ) return b; // equivalent to returning a when b % a == 0  
    else return gcd( b % a, a );  
}
```

A running example of Euclid's algorithm

Function gcd()

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Evaluation of gcd(477,246)

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```
gcd(477, 246)  
    gcd(231, 246)
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gcd(477, 246)  
    gcd(231, 246)  
        gcd(15, 231)
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Evaluation of gcd(477, 246)

```
gcd(477, 246)  
    gcd(231, 246)  
        gcd(15, 231)  
            gcd(6, 15)
```

A running example of Euclid's algorithm

Function gcd()

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int gcd (int a, int b) {  
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}
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Evaluation of gcd(477, 246)

```
gcd(477, 246)  
    gcd(231, 246)  
        gcd(15, 231)  
            gcd(6, 15)  
                gcd(3, 6)
```

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Function gcd()

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    if ( a == 0 ) return b; // equivalent to returning a when b % a == 0  
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Evaluation of gcd(477,246)

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gcd(477, 246)  
    gcd(231, 246)  
        gcd(15, 231)  
            gcd(6, 15)  
                gcd(3, 6)  
                    gcd(0, 3)
```


A running example of Euclid's algorithm

Function gcd()

```
int gcd (int a, int b) {  
    if ( a == 0 ) return b; // equivalent to returning a when b % a == 0  
    else return gcd( b % a, a );  
}
```

Evaluation of gcd(477,246)

```
gcd(477, 246)  
    gcd(231, 246)  
        gcd(15, 231)  
            gcd(6, 15)  
                gcd(3, 6)  
                    gcd(0, 3)  
gcd(477, 246) == 3
```

Divide-and-conquer algorithms

Solve a problem recursively, applying three steps at each level of recursion

Divide the problem into a number of subproblems that are smaller instances of the same problem

Conquer the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.

Combine the solutions to subproblems into the solution for the original problem

Binary Search

```
// assuming a is sorted, return index of array containing the key,  
// among a[start...end]. Return -1 if no key is found  
int binarySearch(std::vector<int>& a, int key, int start, int end) {  
    if ( start > end ) return -1; // search failed  
    int mid = (start+end)/2;  
    if ( key == a[mid] ) return mid; // terminate if match is found  
    if ( key < a[mid] ) // divide the remaining problem into half  
        return binarySearch(a, key, start, mid-1);  
    else  
        return binarySearch(a, key, mid+1, end);  
}
```

Recursive Maximum

```
// find maximum within an a[start..end]
int findMax(std::vector<int>& a, int start, int end) {
    if ( start == end ) return a[start]; // conquer small problem directly
    else {
        int mid = (start+end)/2;
        int leftMax = findMax(a,start,mid); // divide the problem into half
        int rightMax = findMax(a,mid+1,end);
        return ( leftMax > rightMax ? leftMax : rightMax ); // combine solutions
    }
}
```

Merge Sort

Divide and conquer algorithm

Divide Divide the n element sequence to be sorted into two subsequences of $n/2$ elements each

Conquer Sort the two subsequences recursively using merge sort

Combine Merge the two sorted subsequences to produce the sorted answer

<http://www.sorting-algorithms.com/merge-sort>

mergeSort.cpp - main()

```
#include <iostream>
#include <vector>
#include <climits>
void mergeSort(std::vector<int>& a, int p, int r); // defined later
int main(int argc, char** argv) {
    int tok;
    std::vector<int> v;
    if ( argc > 1 ) {
        std::ifstream fin(argv[1]);
        while( fin >> tok ) { v.push_back(tok); }
        fin.close();
    }
    else {
        while( std::cin >> tok ) { v.push_back(tok); }
    }
    mergeSort(v,0,(int)v.size()-1); // same as before except for this line
    for(int i=0; i < v.size(); ++i) {
        std::cout << v[i] << std::endl;
    }
    return 0;
}
```

mergeSort.cpp - merge() function

```
// merge piecewise sorted a[p..q-1] a[q..r] into a sorted a[p..r]
void merge(std::vector<int>& a, int p, int q, int r) {
    std::vector<int> aL, aR; // copy a[p..q-1] to aL and a[q..r] to aR
    for(int i=p; i < q; ++i) aL.push_back(a[i]);
    for(int i=q; i <= r; ++i) aR.push_back(a[i]);
    aL.push_back(INT_MAX); // append additional value to avoid out-of-bound
    aR.push_back(INT_MAX);
    // scan sorted aL and aR separately, taking the minimum between the two
    for(int k=p, i=0, j=0; k <= r; ++k) {
        if ( aL[i] < aR[j] ) { a[k] = aL[i]; ++i; }
        else { a[k] = aR[j]; ++j; }
    }
}
```

mergeSort.cpp - mergeSort() function

```
void mergeSort(std::vector<int>& a, int p, int r) {
    if ( p < r ) {          // skip if the element size is one or less
        int q = (p+r+1)/2; // find a point to divide the problem
        mergeSort(a, p, q-1); // divide-and-conquer
        mergeSort(a, q, r); // divide-and-conquer
        merge(a, p, q, r); // combine the solutions
    }
}
```


Time Complexity of Merge Sort

If $n = 2^m$

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ 2T(n/2) + cn & \text{if } n > 1 \end{cases}$$

$$T(n) = \sum_{i=1}^m cn = cmn = cn \log_2(n) = \Theta(n \log_2 n)$$

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$$T(n) = \sum_{i=1}^m cn = cmn = cn \log_2(n) = \Theta(n \log_2 n)$$

For arbitrary n

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + cn & \text{if } n > 1 \end{cases}$$

$$cn \lfloor \log_2 n \rfloor \leq T(n) \leq cn \lceil \log_2 n \rceil$$

$$T(n) = \Theta(n \log_2 n)$$

Running time comparison

Running example with 100,000 elements (in UNIX or MacOS)

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user@host:~/> time cat src/sample.input.txt | src/insertionSort > /dev/null
real 1m8.795s
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```

```
user@host:~/> time cat src/sample.input.txt | src/mergeSort > /dev/null
real 0m0.898s
user 0m0.755s
sys 0m0.131s
```

Quicksort

Quicksort Overview

- Worse-case time complexity is $\Theta(n^2)$
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- But in practice mostly performs the best

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Divide and conquer algorithm

Divide Partition (rearrange) the array $A[p..r]$ into two subarrays

- Each element of $A[p..q-1] \leq A[q]$
- Each element of $A[q+1..r] \geq A[q]$

Compute the index q as part of this partitioning procedure

Conquer Sort the two subarrays by recursively calling quicksort

Combine Because the subarrays are already sorted, no work is needed to combine them. The entire array $A[p..r]$ is now sorted

Quicksort Algorithm

Algorithm QUICKSORT

Data: array A and indices p and r

Result: $A[p..r]$ is sorted

if $p < r$ **then**

$q = \text{PARTITION}(A, p, r);$

$\text{QUICKSORT}(A, p, q - 1);$

$\text{QUICKSORT}(A, q + 1, r);$

end

Quicksort Algorithm

Algorithm PARTITION

Data: array A and indices p and r

Result: Returns q such that $A[p..q-1] \leq A[q] \leq A[q+1..r]$

$x = A[r];$

$i = p - 1;$

for $j = p$ **to** $r - 1$ **do**

if $A[j] \leq x$ **then**

$i = i + 1;$

 EXCHANGE($A[i], A[j]$);

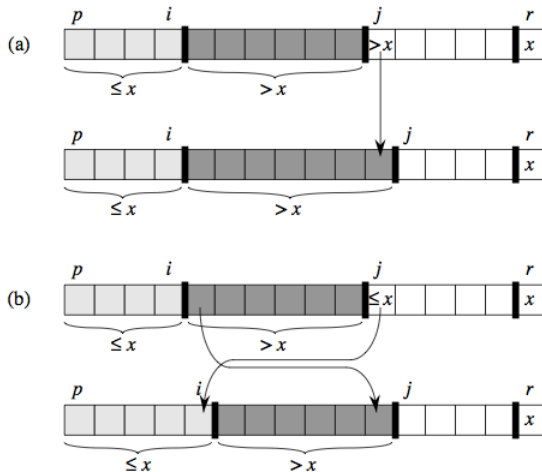
end

end

EXCHANGE($A[i + 1], A[r]$);

return $i + 1;$

How PARTITION Algorithm Works



Implementation of QUICKSORT Algorithm

```
// quickSort function
// The main function is the same to mergeSort.cpp except for the function name
void quickSort(std::vector<int>& A, int p, int r) {
    if ( p < r ) { // immediately terminate if subarray size is 1
        int piv = A[r]; // take a pivot value
        int i = p-1;    // p-i-1 is the # elements < piv among A[p..j]
        int tmp;
        for(int j=p; j < r; ++j) {
            if ( A[j] < piv ) { // if smaller value is found, increase q (=i+1)
                ++i;
                tmp = A[i]; A[i] = A[j]; A[j] = tmp; // swap A[i] and A[j]
            }
        }
        A[r] = A[i+1]; A[i+1] = piv; // swap A[i+1] and A[r]
        quickSort(A, p, i);
        quickSort(A, i+2, r);
    }
}
```

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```

```
user@host:~/> time cat src/sample.input.txt | src/mergeSort > /dev/null
real 0m0.898s
user 0m0.755s
sys 0m0.131s
```

```
user@host:~/> time cat src/sample.input.txt | src/quickSort > /dev/null
real 0m0.427s
user 0m0.285s
sys 0m0.129s
```

Reading Material

- CLRS Chapter 2
- CLRS Chapter 3
- CLRS Chapter 4
- CLRS Chapter 7