Biostatistics 615/815 Lecture 5: More on STLs, and Divide and Conquer Algorithms

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Announcements

- Homework #1 is due, late submission will be better than no submission
- For 815 project, rank your preference by Friday 11:59pm
- Utilize office hours (9:30AM-12:30PM Friday) for further questions
- When classroom is full, the seating priority should be given to enrolled students

Recap on Classes

```
#include <iostream>
class Point {
public:
  int x, y;
  Point(int px, int py) : x(px), y(py) {std::cout << "Point::Point(int,int)" << std::endl;}
  Point() : x(0), y(0) { std::cout << "Point::Point()" << std::endl; }
  void print() { std::cout << "(" << x << "," << y << ")" << std::endl; }
};
int main(int argc, char** argv) { // what happens at each line?
  Point p1;                   // Point::Point()
  Point p2(3,4);              // Point::Point(int,int)
  Point* pp1 = new Point(5,12); // Point::Point(int,int)
  Point* pp2 = new Point[5];  // Point::Point() (5 times)
  p1.print();                 // (0,0)
  p2.print();                 // (3,4)
  pp1->print();               // (3,4)
  pp2->print();               // (0,0)
  pp2[5].print();             // (0,0)
  delete pp1;
  delete [] pp2;
  return 0;
}
```


Recap on STL : vector

```
#include <iostream>
#include <vector>
int main(int argc, char** argv) {
  // vector examples
  std::vector<int> v;
  v.push_back(10);                 // v contains {10}
  v.push_back(5);                  // v contains {10,5}
  v.insert(v.begin(),7);           // v contains {7,10,5}
  v[2] = 1;                        // v contains {7,10,1};
  std::sort(v.begin(),v.end());    // v contains {1,7,10};
  std::cout << v[1] << std::endl; // prints 7
  return 0;
}
```

Recap on STL : string and map

```
#include <string>
#include <map>
int main(int argc, char** argv) {
  // string examples
  std::string s("hell");
  std::cout << s << std::endl;   // prints "hell"
  s += "o";                      // use '+' for concatenation
  std::cout << s << std::endl;   // prints "hello"
  s[0] = 'j';                    // access & assign indiviual character via []
  std::cout << s << std::endl;   // prints "jello"
  std::sort(s.begin(), s.end()); // string is a vector of characters
  std::cout << s << std::endl;   // prints "ejllo";
  // map examples
  std::map<std::string,int> points;
  points["Carlo Sidore"] = 100;
  points["Goncalo Abecasis"] = 99;
  points["Hyun Min Kang"] = 50;
  std::cout << points["Carlo Sidore"] << std::endl; // prints 100
  std::cout << points["Goncalo Abecasis"] << std::endl;     // prints 99
  std::cout << points["Hyun Min Kim"] << std::endl; // nonexistent key; prints default value
}
```


Reading from Files : stdSort.cpp

```
#include <iostream>
#include <fstream>
#include <vector>
int main(int argc, char** argv) { // sorting software using STL
  int tok;
  std::vector<int> v;
  if ( argc > 1 ) { // if argument is given, read from file
    std::ifstream fin(argv[1]);
    while( fin >> tok ) { v.push_back(tok); }
    fin.close();
  }
  else { // read from standard input if no argument is specified
    while( std::cin >> tok ) { v.push_back(tok); }
  }
  std::sort(v.begin(), v.end()); // Sort using the algorithm in STL
  for(int i=0; i < v.size(); ++i) {
    std::cout << v[i] << std::endl; // print out the content
  }
  return 0;
}
```


Reading from Files : insertionSort.cpp

```
#include <iostream>
#include <fstream>
#include <vector>
void insertionSort(std::vector<int>& v); // insertionSort as defined before
int main(int argc, char** argv) {
  int tok;
  std::vector<int> v;
  if ( argc > 1 ) {
    std::ifstream fin(argv[1]);
    while( fin >> tok ) { v.push_back(tok); }
    fin.close();
  }
  else {
    while( std::cin >> tok ) { v.push_back(tok); }
  }
  insertionSort(v); // differs from stdSort in only this part
  for(int i=0; i < v.size(); ++i) {
    std::cout << v[i] << std::endl;
  }
  return 0;
}
```


Running time comparison

Running example with 100,000 elements (in UNIX or MacOS)

```
user@host:~/> time cat src/sample.input.txt | src/stdSort > /dev/null
real 0m0.430s
user 0m0.281s
sys 0m0.130s

user@host:~/> time cat src/sample.input.txt | src/insertionSort > /dev/null
real 1m8.795s
user 1m8.181s
sys 0m0.206s
```

Recursion

Defintion of recursion

Recursion See "Recursion".

Another defintion of recursion

Recursion If you still don't get it, see: "Recursion"

Key components of recursion

- A function that is part of its own definition
- Terminating condition (to avoid infinite recursion)

Example of recursion

Factorial

```
int factorial(int n) {
  if ( n == 0 )
    return 1;
  else
    return n * factorial(n-1); // tail recursion - can be transformed into loop
}
```

towerOfHanoi

```
void towerOfHanoi(int n, int s, int i, int d) { // n disks, from s to d via i
  if ( n > 0 ) {
    towerOfHanoi(n-1,s,d,i); // recursively move n-1 disks from s to i
    // Move n-th disk from s to d
    std::cout << "Disk " << n << " : " << s << " -> " << d << std::endl;
    towerOfHanoi(n-1,i,s,d); // recursively move n-1 disks from i to d
  }
}
```


Euclid's algorithm

Algorithm GCD

Data: Two integers a and b
Result: The greatest common divisor (GCD) between a and b
if a *divides* b **then**
  return *a*
else
  Find the largest integer t such that $at + r = b$;
  return $\text{GCD}(r,a)$
end

Function gcd()

```
int gcd (int a, int b) {
  if ( a == 0 )  return b; // equivalent to returning a when b % a == 0
  else           return gcd( b % a, a );
}
```


A running example of Euclid's algorithm

Function gcd()

```
int gcd (int a, int b) {
  if ( a == 0 )  return b; // equivalent to returning a when b % a == 0
  else           return gcd( b % a, a );
}
```

Evaluation of gcd(477,246)

```
gcd(477, 246)
    gcd(231, 246)
        gcd(15, 231)
            gcd(6, 15)
                gcd(3, 6)
                    gcd(0, 3)
gcd(477, 246) == 3
```

Divide-and-conquer algorithms

Solve a problem recursively, applying three steps at each level of recursion

- **Divide** the problem into a number of subproblems that are smaller instances of the same problem
- **Conquer** the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.
- **Combine** the solutions to subproblems into the solution for the original problem

Binary Search

```
// assuming a is sorted, return index of array containing the key,
// among a[start...end]. Return -1 if no key is found
int binarySearch(std::vector<int>& a, int key, int start, int end) {
  if ( start > end ) return -1; // search failed
  int mid = (start+end)/2;
  if ( key == a[mid] ) return mid; // terminate if match is found
  if ( key < a[mid] ) // divide the remaining problem into half
    return binarySearch(a, key, start, mid-1);
  else
    return binarySearch(a, key, mid+1, end);
}
```


Recursive Maximum

```
// find maximum within an a[start..end]
int findMax(std::vector<int>& a, int start, int end) {
  if ( start == end ) return a[start];  // conquer small problem directly
  else {
    int mid = (start+end)/2;
    int leftMax = findMax(a,start,mid);  // divide the problem into half
    int rightMax = findMax(a,mid+1,end);
    return ( leftMax > rightMax ? leftMax : rightMax ); // combine solutions
  }
}
```


Merge Sort

Divide and conquer algorithm

- **Divide** Divide the n element sequence to be sorted into two subsequences of n/2 elements each
- **Conquer** Sort the two subsequences recursively using merge sort
- **Combine** Merge the two sorted subsequences to produce the sorted answer

http://www.sorting-algorithms.com/merge-sort

mergeSort.cpp - main()

```
#include <iostream>
#include <vector>
#include <climits>
void mergeSort(std::vector<int>& a, int p, int r); // defined later
int main(int argc, char** argv) {
  int tok;
  std::vector<int> v;
  if ( argc > 1 ) {
    std::ifstream fin(argv[1]);
    while( fin >> tok ) { v.push_back(tok); }
    fin.close();
  }
  else {
    while( std::cin >> tok ) { v.push_back(tok); }
  }
  mergeSort(v,0,(int)v.size()-1); // same as before except for this linee
  for(int i=0; i < v.size(); ++i) {
    std::cout << v[i] << std::endl;
  }
  return 0;
}
```


Introduction STL Recursion Gcd Divide and Conquer MergeSort Quicksort

mergeSort.cpp - merge() function

```
// merge piecewise sorted a[p..q-1] a[q..r] into a sorted a[p..r]
void merge(std::vector<int>& a, int p, int q, int r) {
  std::vector<int> aL, aR; // copy a[p..q-1] to aL and a[q..r] to aR
  for(int i=p; i < q; ++i) aL.push_back(a[i]);
  for(int i=q; i <= r; ++i) aR.push_back(a[i]);
  aL.push_back(INT_MAX); // append additional value to avoid out-of-bound
  aR.push_back(INT_MAX);
  // scan sorted aL and aR separately, taking the minimum between the two
  for(int k=p, i=0, j=0; k <= r; ++k) {
    if ( aL[i] < aR[j] ) { a[k] = aL[i]; ++i; }
    else { a[k] = aR[j]; ++j; }
  }
}
```


Introduction STL Recursion Gcd Divide and Conquer MergeSort Quicksort

mergeSort.cpp - mergeSort() function

```
void mergeSort(std::vector<int>& a, int p, int r) {
  if ( p < r ) {         // skip if the element size is one or less
    int q = (p+r+1)/2;     // find a point to divide the problem
    mergeSort(a, p, q-1);    // divide-and-conquer
    mergeSort(a, q, r);  // divide-and-conquer
    merge(a, p, q, r);     // combine the solutions
  }
}
```


Introduction STL Recursion Gcd Divide and Conquer MergeSort Quicksort

Time Complexity of Merge Sort

If $n = 2^m$

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ 2T(n/2) + cn & \text{if } n > 1 \end{cases}$$

$$T(n) = \sum_{i=1}^{m} cn = cmn = cn\log_2(n) = \Theta(n \log_2 n)$$

For arbitrary n

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + cn & \text{if } n > 1 \end{cases}$$

$$cn\lfloor \log_2 n \rfloor \leq T(n) \leq cn\lceil \log_2 n \rceil$$

$$T(n) = \Theta(n \log_2 n)$$

Running time comparison

Running example with 100,000 elements (in UNIX or MacOS)

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real 0m0.430s
user 0m0.281s
sys 0m0.130s

user@host:~/> time cat src/sample.input.txt | src/insertionSort > /dev/null
real 1m8.795s
user 1m8.181s
sys 0m0.206s

user@host:~/> time cat src/sample.input.txt | src/mergeSort > /dev/null
real 0m0.898s
user 0m0.755s
sys 0m0.131s
```


Quicksort

Quicksort Overview

- Worse-case time complexity is $\Theta(n^2)$
- Expected running time is $\Theta(n \log_2 n)$.
- But in practice mostly performs the best

Divide and conquer algorithm

Divide Parition (rearrange) the array $A[p..r]$ into two subarrays
- Each element of $A[p..q-1] \leq A[q]$
- Each element of $A[q+1..r] \geq A[q]$

Compute the index q as part of this partitioning procedure

Conquer Sort the two subarrays by recursively calling quicksort

Combine Because the subarrays are already sorted, no work is needed to combine them. The entire array $A[p..r]$ is now sorted

Quicksort Algorithm

Algorithm QUICKSORT

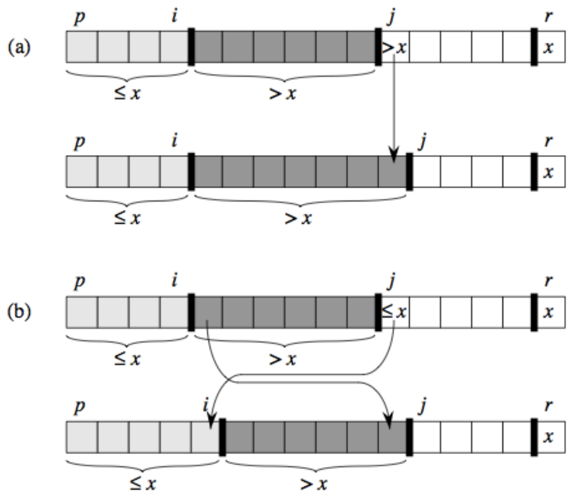
Data: array A and indices p and r
Result: $A[p..r]$ is sorted
if $p < r$ **then**
  q = PARTITION(A,p,r);
  QUICKSORT(A,p,$q-1$);
  QUICKSORT(A,$q+1$,r);
end

Quicksort Algorithm

Algorithm PARTITION

Data: array A and indices p and r
Result: Returns q such that $A[p..q-1] \leq A[q] \leq A[q+1..r]$
$x = A[r]$;
$i = p - 1$;
for $j = p$ **to** $r - 1$ **do**
  if $A[j] \leq x$ **then**
    $i = i + 1$;
    EXCHANGE($A[i]$,$A[j]$);
  end
end
EXCHANGE($A[i+1]$,$A[r]$);
return $i + 1$;

How PARTITION Algorithm Works



Implementation of QUICKSORT Algorithm

```
// quickSort function
// The main function is the same to mergeSort.cpp except for the function name
void quickSort(std::vector<int>& A, int p, int r) {
  if ( p < r ) { // immediately terminate if subarray size is 1
    int piv = A[r];  // take a pivot value
    int i = p-1;     // p-i-1 is the # elements < piv among A[p..j]
    int tmp;
    for(int j=p; j < r; ++j) {
      if ( A[j] < piv ) { // if smaller value is found, increase q (=i+1)
        ++i;
        tmp = A[i]; A[i] = A[j]; A[j] = tmp; // swap A[i] and A[j]
      }
    }
    A[r] = A[i+1]; A[i+1] = piv;             // swap A[i+1] and A[r]
    quickSort(A, p, i);
    quickSort(A, i+2, r);
  }
}
```


Running time comparison

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real 1m8.795s
user 1m8.181s
sys 0m0.206s

user@host:~/> time cat src/sample.input.txt | src/mergeSort > /dev/null
real 0m0.898s
user 0m0.755s
sys 0m0.131s

user@host:~/> time cat src/sample.input.txt | src/quickSort > /dev/null
real 0m0.427s
user 0m0.285s
sys 0m0.129s
```


Reading Material

- CLRS Chapter 2
- CLRS Chapter 3
- CLRS Chapter 4
- CLRS Chapter 7