Biostatistics 615/815 Lecture 7: Elementary Data Structures

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Elementary data structure

Container

A container T is a generic data structure which supports the following three operation for an object x.

- $\textsc{Search}(T, x)$
- $\textsc{Insert}(T, x)$
- $\textsc{Delete}(T, x)$

Possible types of container

- Arrays
- Linked lists
- Trees
- Hashes

Average time complexity of container operations

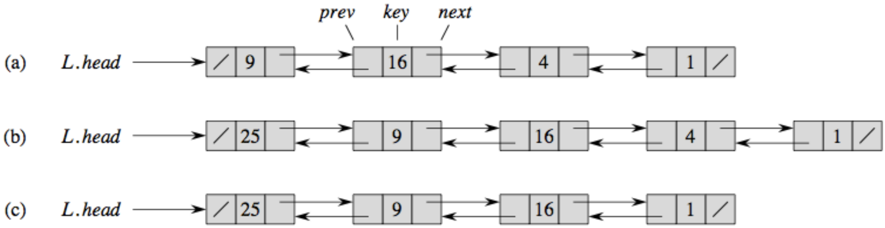
	SEARCH	INSERT	DELETE
Array	$\Theta(n)$	$\Theta(1)$	$\Theta(n)$
SortedArray	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n)$
List	$\Theta(n)$	$\Theta(1)$	$\Theta(n)$
Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

- Array or list is simple and fast enough for small-sized data
- Tree is easier to scale up to moderate to large-sized data
- Hash is the most robust for very large datasets

Linked List

- A data structure where the objects are arranged in linear order
- Each object contains the pointer to the next object
- Objects do not exist in consecutive memory space
  - No need to shift elements for insertions and deletions
  - No need to allocate/reallocate the memory space
  - Need to traverse elements one by one
  - Likely inefficient than `Array` in practice because data is not necessarily localized in memory
- Variants in implementation
  - (Singly-) linked list
  - Doubly-linked list

Example of a linked list



- Example of a doubly-linked list
- Singly-linked list if `prev` field does not exist

Implementation of singly-linked list

myList.h

```
#include "myListNode.h"
template <class T>
class myList {
protected:
  myListNode<T>* head; // list only contains the pointer to head
  myList(myList& a) {};    // prevent copying
public:
  myList() : head(NULL) {} // initially header is NIL
  ~myList();
  void insert(T x); // insert an element x
  int search(T x);  // search for an element x and return its location
  bool remove(T x); // delete a particular element
};
```

List implementation : class myListNode

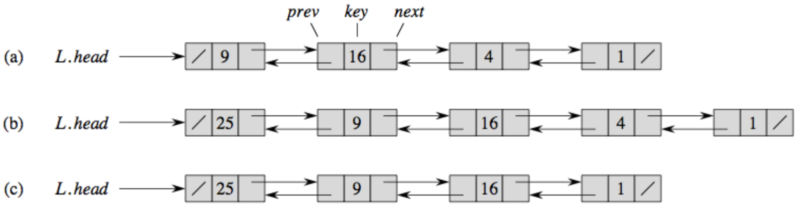
myListNode.h

```
// myListNode class is only accessible from myList class
template<class T>
class myListNode {
protected:
  T value;           // the value of each element
  myListNode<T>* next;  // pointer to the next element
  myListNode(T v, myListNode<T>* n) : value(v), next(n) {} // constructor
  ~myListNode();
  int search(T x, int curPos);
  myListNode<T>* remove(T x, myListNode<T>*& prevNext);
  template <class S> friend class myList; // allow full access to myList class
};
```

Inserting an element to a list

myList.h

```
template <class T>
void myList<T>::insert(T x) {
  // create a new node, and make them head
  // and assign the original head to head->next
  head = new myListNode<T>(x, head);
}
```



Destructor is required because new was used

myList.h

```
template <class T>
myList<T>::~myList() {
  if ( head != NULL ) {
    delete head;    // delete dependent objects before deleting itself
  }
}
```

myListNode.cpp

```
template <class T>
myListNode<T>::~myListNode() {
  if ( next != NULL ) {
    delete next;  // recursively calling destructor until the end of the list
  }
}
```

Searching an element from a list

myList.h

```
template <class T>
int myList<T>::search(T x) {
  if ( head == NULL )  return -1;  // NOT_FOUND if empty
  else return head->search(x, 0);  // search from the head node
}
```

myListNode.cpp

```
template <class T>
// search for element x, and the current index is curPos
int myListNode<T>::search(T x, int curPos) {
  if ( value == x )          return curPos; // if found return current index
  else if ( next == NULL ) return -1;      // NOT_FOUND if reached end-of-list
  else return next->search(x, curPos+1);  // recursive call until terminates
}
```

Removing an element from a list

myList.h

```
template <class T>
bool myList<T>::remove(T x) {
  if ( head == NULL )
    return false;     // NOT_FOUND if the list is empty
  else {
    // call head->remove will return the object to be removed
    myListNode<T>* p = head->remove(x, head);
    if ( p == NULL ) { // if NOT_FOUND return false
      return false;
    }
    else {            // if FOUND, delete the object before returning true
      delete p;
      return true;
    }
  }
}
```

Removing an element from a list

myListNode.h

```
template <class T>
// pass the pointer to [prevElement->next] so that we can change it
myListNode<T>* myListNode<T>::remove(T x, myListNode<T>*& prevNext) {
  if ( value == x ) {   // if FOUND
    prevNext = next;    // *pPrevNext was this, but change to next
    next = NULL;        // disconnect the current object from the list
    return this;        // and return it so that it can be destroyed
  }
  else if ( next == NULL ) {
    return NULL;        // return NULL if NOT_FOUND
  }
  else {
    return next->remove(x, next); // recursively call on the next element
  }
}
```

Summary - Linked List

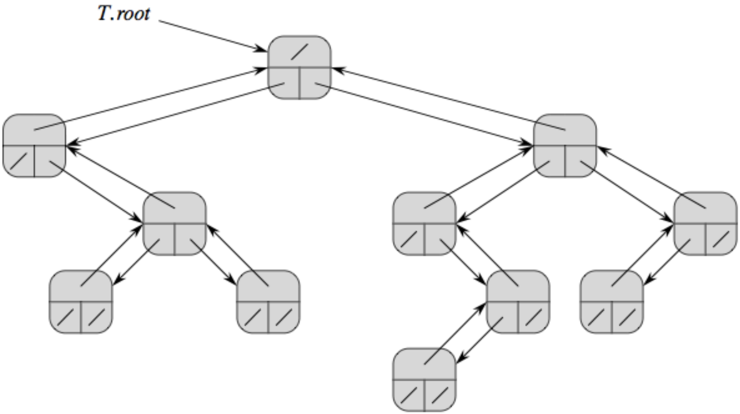
- Class Structure
  - `myList` class to keep the head node
  - `myListNode` class to store key and pointer to next node
- `Insert` algorithm : Create a new node as a head node
- `Search` algorithm
  - Return the index if key matches
  - Otherwise, advance to the next node
- `Remove` algorithm :
  - Search the element
  - Make the previous node points to the next node
  - Remove the element from the list and destroy it.
- Q: What are the advantages and disadvantages between Array and List?
-

Binary search tree

Data structure

- The tree contains a root node
- Each node contains
  - Pointers to `left` and `right` children
  - Possibly a pointer to its parent
  - And a key value
- Sorted : `left.key` $\leq$ `key` $\leq$ `right.key`
- Average $\Theta(\log n)$ complexity for insert, search, remove operations

An example binary search tree



Key algorithms

$\textsc{Insert}(node, x)$

1. If the $node$ is empty, create a leaf node with value x and return
2. If $x < node.key$, $\textsc{Insert}(node.left, x)$
3. Otherwise, $\textsc{Insert}(node.right, x)$

$\textsc{Search}(node, x)$

1. If $node$ is empty, return $-\infty$
2. If $node.key == x$, return size$(node.left)$
3. If $x < node.key$, return $\textsc{Search}(node.left, x)$
4. If $x > node.key$, return $\textsc{Search}(node.right, x) + 1 +$ size$(node.left)$

Key algorithms

REMOVE($node, x$)

1. If $node.key == x$
   1. If the node is leaf, remove the node
   2. If the node only has left child, replace the current node to the left child
   3. If the node only has right child, replace the current node to the right child
   4. Otherwise, pick either maximum among left sub-tree or minimum among right subtree and substitute the node into the current node
2. If $x < node.key$
   1. Call REMOVE($node.left, x$) if $node.left$ exists
   2. Otherwise, return NOTFOUND
3. If $x > node.key$
   1. Call REMOVE($node.right, x$) if $node.right$ exists
   2. Otherwise, return NOTFOUND

Implementation of binary search tree

myTree.h

```
template <class T>
class myTree {
protected:
  myTreeNode<T>* pRoot;      // tree contains pointer to root
  myTree(myTree& a) {};      // prevent copying
public:
  myTree() : pRoot(NULL) {} // initially root is empty
  ~myTree() { if ( pRoot != NULL ) delete pRoot; } // destructor
  void insert(T x);
  int search(T x);
  bool remove(T x);
};
```

Implementation of binary search tree

myTreeNode.h

```
template <class T>
class myTreeNode {
  T value;   // key value
  int size;  // total number of nodes in the subtree
  myTreeNode<T>* left;  // pointer to the left subtree
  myTreeNode<T>* right; // pointer to the right subtree
  myTreeNode(T x, myTreeNode<T>* l, myTreeNode<T>* r); // constructors
  ~myTreeNode();            // destructors
  void insert(T x); // insert an element
  int search(T x);
  myTreeNode<T>* remove(T x, myTreeNode<T>** ppSelf);
  T getMax();               // maximum value in the subtree
  T getMin();               // minimum value in the subtree
};
```

Binary search tree : Constructors and Destructors

myTreeNode.h

```
template<class T>
myTreeNode<T>::myTreeNode(T x, myTreeNode<T>* l, myTreeNode<T>* r) : value(x), siz

template<class T>
myTreeNode<T>::~myTreeNode() {
  // remove child nodes before removing the node itself
  if ( left != NULL ) delete left;
  if ( right != NULL ) delete right;
}
```

Binary search tree : INSERT

myTree.h

```
template <class T>
void myTree<T>::insert(T x) {
  if ( pRoot == NULL )
     pRoot = new myTreeNode<T>(x,NULL,NULL);  // create a root if empty
  else
     pRoot->insert(x);                        // insert to the root
}
```

Binary search tree : INSERT

myTreeNode.h

```
template <class T>
void myTreeNode<T>::insert(T x) {
  if ( x < value ) {     // if key is small, insert to the left subtree
    if ( left == NULL )
      left = new myTreeNode<T>(x,NULL,NULL); // create if doesn't exist
    else
      left->insert(x);
  }
  else {                  // otherwise, insert to the right subtree
    if ( right == NULL )
      right = new myTreeNode<T>(x,NULL,NULL);
    else
      right->insert(x);
  }
  ++size;
}
```

Binary search tree : SEARCH

myTree.h

```
template <class T>
int myTree<T>::search(T x) {
  if ( pRoot == NULL )
    return -1;
  else
    return pRoot->search(x);
}
```

Binary search tree : SEARCH

myTreeNode.h

```
template <class T>   // return the 0-based rank of the value x
int myTree<T>::search(T x) {
  if ( x == value ) {           // if key matches to the value
    if ( left == NULL )
      return 0;                 // return 0 if there is no smaller element
    else
      return left->size;        // return # of left-subtree otherwise
  }
  else if ( x < value ) {       // recursively call the function to left subtree
    if ( left == NULL )
      return -1;
    else
    return left->search(x);
  }
```

Binary search tree : SEARCH

myTreeNode.h (cont'd)

```
  else { // when x > value, [#leftSubtree]+1 should be added
    if ( right == NULL )
      return -1;
    else {
      int r = right->search(x);
      if ( r < 0 ) return -1;
      else if ( left == NULL ) return ( 1 + r );
      else return ( left->size + 1 + r );
    }
  }
}
```

Binary search tree : REMOVE

myTree.h

```
template <class T>
bool myTree<T>::remove(T x) {
  if ( pRoot == NULL ) {
    return false;
  }
  else {
    myTreeNode<T>* p = pRoot->remove(x, pRoot);
    if ( p != NULL ) {  // if an object was removed
      delete p;         // destroy the object
      return true;      // and return true
    }
    else {
      return false;     // return false if the object was not found
    }
  }
}
```

Binary search tree : Remove

myTreeNode.h

```
template <class T>
myTreeNode<T>* myTreeNode<T>::remove(T x, myTreeNode<T>*& pSelf) {
  if ( x == value ) {  // key was found
    if ( ( left == NULL ) && ( right == NULL ) ) { // no child
      pSelf = NULL;
      return this;
    }
    else if ( left == NULL ) { // only left is NULL
      pSelf = right;
      right = NULL;
      return this;
    }
    else if ( right == NULL ) {  // only right is NULL
      pSelf = left;
      left = NULL;
      return this;
    } // ....
```

Binary search tree : Remove (cont'd)

myTreeNode.h

```
    else { // neither left nor right is NULL
      // choose which subtree to delete
      myTreeNode<T>* p;
      if ( left->size > right->size ) { // if left subtree is larger
        T m = left->getMax();        // copy the largest value among them
        p = left->remove(m, left);  // to current node, and delete the node
        value = m;
      }
      else {
        T m = right->getMin();        // copy smallest value among them
        p = right->remove(m, right); // to current node, and delete the node
        value = m;
      }
      return p;
    }
  }
  // ....
```

Binary search tree : REMOVE (cont'd)

myTreeNode.h

```
  else if ( x < value ) {
    if ( left == NULL )
      return NULL;
    else
      return left->remove(x, left);
  }
  else { // x > value
    if ( right == NULL )
      return NULL;
    else
      return right->remove(x, right);
  }
}
```

Binary search tree : GETMAX and GETMIN

myTreeNode.h

```
template <class T>
T myTreeNode<T>::getMax() {  // return the largest value
  if ( right == NULL ) return value;
  else return right->getMax();
}

template <class T>
T myTreeNode<T>::getMin() {  // return the smallest value
  if ( left == NULL ) return value;
  else return left->getMin();
}
```

If you want to print a tree...

myTreeNode.h

```
template <class T> void myTreeNode<T>::print() {
  std::cout << "[ ";
  if ( left != NULL ) left->print();
  else std::cout << "[ NULL ]";
  std::cout << " , (" << value << "," << size << ") , ";
  if ( right != NULL ) right->print();
  else std::cout << "[ NULL ]";
  std::cout << " ]";
}
```

myTree.h

```
template <class T> void myTree<T>::print() {
  if ( pRoot != NULL ) pRoot->print();
  else std::cout << "(EMPTY TREE)";
  std::cout << std::endl;
}
```

Summary - Binary Search Tree

- Key Features
  - Fast insertion, search, and removal
  - Implementation is much more complicated
- Class Structure
  - `myTree` class to keep the root node
  - `myTreeNode` class to store key and up to two children
- Key Algorithms

  Insert : Traverse the tree in sorted order and create a new node in the first leaf node.

  Search : Divide-and-conquer algorithms

  Remove : Move the nearest leaf element among the subtree and destroy it.

Two types of containers

Containers for single-valued objects - last lecture

- $\text{INSERT}(T, x)$ - Insert x to the container.
- $\text{SEARCH}(T, x)$ - Returns the location/index/existence of x.
- $\text{REMOVE}(T, x)$ - Delete x from the container if exists
- STL examples include `std::vector`, `std::list`, `std::deque`, `std::set`, and `std::multiset`.

Containers for `(key,value)` pairs - this lecture

- $\text{INSERT}(T, x)$ - Insert $(x.key, x.value)$ to the container.
- $\text{SEARCH}(T, k)$ - Returns the value associated with key k.
- $\text{REMOVE}(T, x)$ - Delete element x from the container if exitst
- Examples include `std::map`, `std::multimap`, and `__gnu_cxx::hash_map`

Direct address tables

An example (key,value) container

- $U = \{0, 1, \cdots, N-1\}$ is possible values of keys (N is not huge)
- No two elements have the same key

Direct address table : a constant-time continaer

Let $T[0, \cdots, N-1]$ be an array space that can contain N objects

- $\textsc{Insert}(T, x)$: $T[x.key] = x$
- $\textsc{Search}(T, k)$: $\textsc{return}\ T[k]$
- $\textsc{Remove}(T, x)$: $T[x.key] = \textsc{Nil}$

Analysis of direct address tables

Time complexity

- Requires a single memory access for each operation
- $O(1)$ - constant time complexity

Memory requirement

- Requires to pre-allocate memory space for any possible input value
- $2^{32} = 4GB\times$(size of data) for 4 bytes (32 bit) key
- $2^{64} = 18EB(1.8 \times 10^7 TB)\times$(size of data) for 8 bytes (64 bit) key
- An infinite amount of memory space needed for storing a set of arbitrary-length strings (or exponential to the length of the string)

Hash Tables

Key features

- $O(1)$ complexity for INSERT, SEARCH, and REMOVE
- Requires large memory space than the actual content for maintainng good performance
- But uses much smaller memory than direct-addres tables

Key components

- Hash function
  - $h(x.key)$ mapping key onto smaller 'addressible' space H
  - Total required memory is the possible number of hash values
  - Good hash function minimize the possibility of key collisions
- Collision-resolution strategy, when $h(k_1) = h(k_2)$.

Summary

Today

- List
- Binary Search Tree
- Direct Address Table
- Introduction to hash table

Next Lecture

- More hash tables
- Dynamic programming