Biostatistics 615/815 Lecture 6: Linear Sorting Algorithms and Elementary Data Structures

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Announcements

A good and bad news

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- Homework #2 will be announced in the next lecture

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815 projects

- 5-6 team pairs in total
- Team assignment will be made during this week
- Each team should set up a meeting with the instructor to kick-start the project

Recap on sorting algorithms : Insertion sort

Algorithm description

1. For each $j \in [2 \cdots n]$, iterate element at indices $j-1, j-2, \cdots 1$.
2. If $A[i] > A[j]$, swap $A[i]$ and $A[j]$
3. If $A[i] <= A[j]$, increase j and go to step 1

Insertion sort is loop invariant

At the start of each iteration, $A[1 \cdots j-1]$ is loop invariant iff:

- $A[1 \cdots j-1]$ consist of elements originally in $A[1 \cdots j-1]$.
- $A[1 \cdots j-1]$ is in sorted order.

Time complexity of Insertion sort

- Worst and average case time-complexity is $\Theta(n^2)$

Recap on sorting algorithms : Mergesort

Algorithm MERGESORT

```
Data: array $A$ and indices $p$ and $r$
Result: $A[p..r]$ is sorted
if $p < r$ then
    $q = \lfloor (p+r)/2 \rfloor$;
    MERGESORT($A$,$p$,$q$);
    MERGESORT($A$,$q+1$,$r$);
    MERGE($A$,$p$,$q$,$r$);
end
```

Time complexity of Mergesort

- Worst and average case time-complexity is $\Theta(n \log n)$

Recap on sorting algorithms : Quicksort

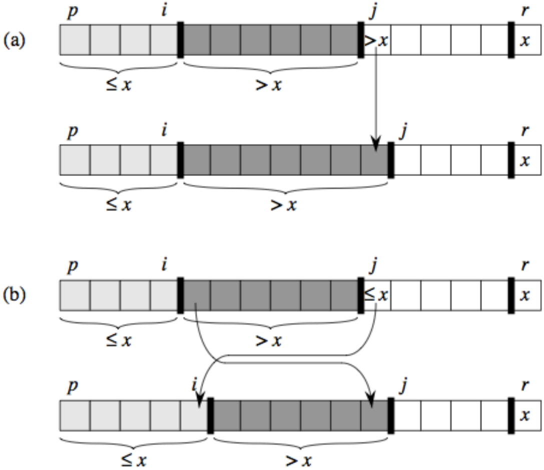
Algorithm QUICKSORT

```
Data: array $A$ and indices $p$ and $r$
Result: $A[p..r]$ is sorted
if $p < r$ then
    $q =$ PARTITION($A$,$p$,$r$);
    QUICKSORT($A$,$p$,$q - 1$);
    QUICKSORT($A$,$q + 1$,$r$);
end
```

Time complexity of Quicksort

- Average case time-complexity is $\Theta(n \log n)$
- Worst case time-complexity is $\Theta(n^2)$, but practically faster than other $\Theta(n \log n)$ algorithms.

How PARTITION algorithm works



Performance of sorting algorithms in practice

Running example with 100,000 elements (in UNIX or MacOS)

```
user@host:~/> time cat src/sample.input.txt | src/stdSort > /dev/null
real 0m0.430s
user 0m0.281s
sys 0m0.130s

user@host:~/> time cat src/sample.input.txt | src/insertionSort > /dev/null
real 1m8.795s
user 1m8.181s
sys 0m0.206s

user@host:~/> time cat src/sample.input.txt | src/mergeSort > /dev/null
real 0m0.898s
user 0m0.755s
sys 0m0.131s

user@host:~/> time cat src/sample.input.txt | src/quickSort > /dev/null
real 0m0.427s
user 0m0.285s
sys 0m0.129s
```

Lower bounds for comparison sorting

CLRS Theorem 8.1

Any comparison-based sort algorithm requires $\Omega(n \log n)$ comparisons in the worst case

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An informal proof

- Any comparison sort algorithm can be represented as a binary decision tree, where each node represents a comparison. Each path from the root to leaf represents possible series of comparisons to sort a sequence.

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- Each leaf of the decision tree represents one of $n!$ possible permutations of input sequences

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- Each leaf of the decision tree represents one of $n!$ possible permutations of input sequences
- We have $n! \leq l \leq 2^h$, where l is the number of leaf nodes, and h is the height of the tree, equivalent to the $\#$ of comparisons.

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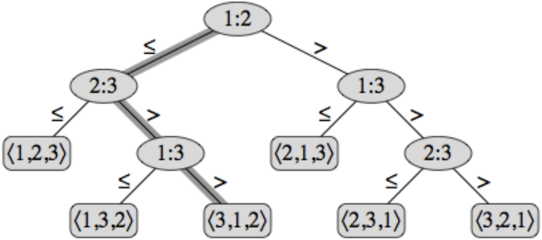
An informal proof

- Any comparison sort algorithm can be represented as a binary decision tree, where each node represents a comparison. Each path from the root to leaf represents possible series of comparisons to sort a sequence.
- Each leaf of the decision tree represents one of $n!$ possible permutations of input sequences
- We have $n! \leq l \leq 2^h$, where l is the number of leaf nodes, and h is the height of the tree, equivalent to the $\#$ of comparisons.
- Then it implies $h \geq \log(n!) = \Theta(n \log n)$

Example decision-tree representing INSERTIONSORT



Finding faster sorting methods

Sorting faster than $\Theta(n \log n)$

- Comparison-based sorting algorithms cannot be faster than $\Theta(n \log n)$
- Sorting algorithms NOT based on comparisons may be faster

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Linear time sorting algorithms

- Counting sort
- Radix sort
- Bucket sort

A linear sorting algorithm : Counting sort

A restrictive input setting

- The input sequences have a finite range with many expected duplication.
- For example, each elements of input sequences is one digit number, and your input sequences are millions.

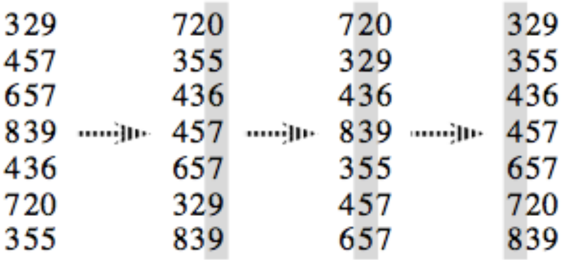
Key idea

1. Scan through each input sequence and count number of occurrences of each possible input value.
2. From the smallest to largest possible input value, output each value repeatedly by its stored count.

Another linear sorting algorithm : Radix sort

Key idea

- Sort the input sequence from the last digit to the first repeatedly using a linear sorting algorithm such as COUNTINGSORT
- Applicable to integers within a finite range



329	720	720	329
457	355	329	355
657	436	436	436
839	457	839	457
436	657	355	657
720	329	457	720
355	839	657	839

Implementing `radixSort.cpp`

```
// use #[radixBits] bits as radix (e.g. hexadecimal if radixBits=4)
void radixSort(std::vector<int>& A, int radixBits, int max) {
  // calculate the number of digits required to represent the maximum number
  int nIter = (int)(ceil(log((double)max)/log(2.)/radixBits));
  int nCounts = (1 << radixBits); // 1<<radixBits == 2^radixBits == # of digits
  int mask = nCounts-1;           // mask for extracting #(radixBits) bits
  std::vector< std::vector<int> > B; // vector of vector, each containing
              // the list of input values containing a particular digit
  B.resize(nCounts);
  for(int i=0; i < nIter; ++i) {
    // initialze each element of B as a empty vector
    for(int j=0; j < nCounts; ++j) { B[j].clear(); }
    // distribute the input sequences into multiple bins, based on i-th digit
    radixSortDivide(A, B, radixBits*i, mask);
    // merge the distributed sequences B into original array A
    radixSortMerge(A, B);
  }
}
```

Implementing `radixSort.cpp`

```
// divide input sequences based on a particular digit
void radixSortDivide(std::vector<int>& A,
             std::vector< std::vector<int> >& B, int shift, int mask) {
  for(int i=0; i < (int)A.size(); ++i) {
    // (A[i]>>shift)&mask takes last [shift .. shift+radixBits-1] bits of A[i]
    B[ (A[i] >> shift) & mask ].push_back(A[i]);
  }
}
// merge the partitioned sequences into single array
void radixSortMerge(std::vector<int>& A, std::vector< std::vector<int> >&B ) {
  for(int i=0, k=0; i < (int)B.size(); ++i) {
    for(int j=0; j < (int)B[i].size(); ++j) {
      A[k] = B[i][j]; // iterate each bin of digit and concatenate all values
      ++k;
    }
  }
}
```

Bitwise operation examples

shift=3, radixBits=1, A[i] = 117

```
117     = 1110101
----------------
117>>3 =     110
mask    =       1
---------------
ret     =       0
```

Bitwise operation examples

shift=3, radixBits=1, A[i] = 117

```
117    = 1110101
----------------
117>>3 =     110
mask   =       1
---------------
ret    =       0
```

shift=3, radixBits=3, A[i] = 117

```
117    = 1110101
----------------
117>>3 =     110
mask   =     111
----------------
ret    =     110
```

Radix sort in practice

```
user@host:~/> time cat src/sample.input.txt | src/stdSort > /dev/null
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user@host:~/> time cat src/sample.input.txt | src/quickSort > /dev/null
real 0m0.427s
user 0m0.285s
sys 0m0.129s

user@host:~/> time cat src/sample.input.txt | src/radixSort 8 > /dev/null
real 0m0.334s
user 0m0.195s
sys 0m0.129s
```

Elementary data structure

Container

A container T is a genetic data structure which supports the following three operation for an object x.

- $\textsc{Search}(T, x)$
- $\textsc{Insert}(T, x)$
- $\textsc{Delete}(T, x)$

Possible types of container

- Arrays
- Linked lists
- Trees
- Hashes

Average time complexity of container operations

	SEARCH	INSERT	DELETE
Array	$\Theta(n)$	$\Theta(1)$	$\Theta(n)$
SortedArray	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n)$
List	$\Theta(n)$	$\Theta(1)$	$\Theta(n)$
Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

- Array or list is simple and fast enough for small-sized data
- Tree is easier to scale up to moderate to large-sized data
- Hash is the most robust for very large datasets

Arrays

Key features

- Stores the data in a consecutive memory space
- Fastest when the data size is small due to locality of data

Using `std::vector` as array

```
std::vector<int> v;  // creates an empty vector
// INSERT : append at the end, O(1)
v.push_back(10);
// SEARCH : find a value scanning from begin to end, O(n)
std::vector<int>::iterator i = std::find(v.begin(), v.end(), 10);
if ( i != v.end() ) { std::cout << "Found " << (*i) << std::endl; }
// DELETE : search first, and delete, O(n)
if ( i != v.end() ) { v.erase(i); } // delete an element
```

Implementing data structure on your own

myArray.h

```
class myArray {
  int* data;
  int size;
  void insert(int x);
  ...
};
```

myArray.cpp

```
#include "myArray.h"
void myArray::insert(int x) { // function body goes here
...
```

Main.cpp

```
#include <iostream>
#include "myArray.h"
int main(int argc, char** argv) {
...
```

Building your program

Individually compile and link

```
user@host:~/> g++ -c myArray.cpp
user@host:~/> g++ -c Main.cpp
user@host:~/> g++ -o myArrayTest Main.o myArray.o
```

Or create a Makefile and just type 'make'

```
all: myArrayTest  # binary name is myArrayTest

myArrayTest: myArray.o Main.o  # link two object files to build binary
    g++ -o myArrayTest myArray.o Main.o  # must start with a tab

Main.o: Main.cpp myArray.h  # compile to build an object file
    g++ -c Main.cpp

myArray.o: myArray.cpp myArray.h # compile to build an object file
    g++ -c myArray.cpp

clean:
    rm *.o myArrayTest
```

Designing a simple array - `myArray.h`

```
// myArray.h declares the interface of the class, and the definition is in myArray
#define DEFAULT_ALLOC 1024
template <class T> // template supporting a generic type
class myArray {
protected:     // member variables hidden from outside
   T *data;    // array of the genetic type
   int size;   // number of elements in the container
   int nalloc; // # of objects allocated in the memory
public:      // abstract interface visible to outside
   myArray();         // default constructor
   ~myArray();        // destructor
   void insert(const T& x); // insert an element x
   int search(const T& x);  // search for an element x and return its location
   bool remove(const T& x); // delete a particular element
};
```

Using a simple array `Main.cpp`

```
#include <iostream>
#include "myArray.h"
int main(int argc, char** argv) {
  myArray<int> A;
  A.insert(10);                 // insert example
  if ( A.search(10) > 0 ) {    // search example
    std::cout << "Found element 10" << std::endl;
  }
  A.remove(10);                 // remove example
  return 0;
}
```

Implementing a simple array `myArray.cpp`

```
template <class T>
myArray<T>::myArray() {  // default constructor
  size = 0;              // array do not have element initially
  nalloc = DEFAULT_ALLOC;
  data = new T[nalloc];  // allocate default # of objects in memory
}

template <class T>
myArray<T>::~myArray() { // destructor
  if ( data != NULL ) {
    delete [] data;      // delete the allocated memory before destroying
  }                      // the object. otherwise, memory leak happens
}
```

myArray.cpp : insert

```
template <class T>
void myArray<T>::insert(const T& x) {
  if ( size >= nalloc ) {  // if container has more elements than allocated
    T* newdata = new T[nalloc*2];   // make an array at doubled size
    for(int i=0; i < nalloc; ++i) {
      newdata[i] = data[i];         // copy the contents of array
    }
    delete [] data;                 // delete the original array
    data = newdata;                 // and reassign data ptr
    nalloc *= 2;                    // double the allocation
  }
  data[size] = x;                   // push back to the last element
  ++size;                           // increase the size
}
```

myArray.cpp : search

```
template <class T>
int myArray<T>::search(const T& x) {
  for(int i=0; i < size; ++i) { // iterate each element
    if ( data[i] == x ) {
      return i;                  // and return index of the first match
    }
  }
  return -1;                     // return -1 if no match found
}
```

myArray.cpp : remove

```
template <class T>
bool myArray<T>::remove(const T& x) {
  int i = search(x);        // try to find the element
  if ( i > 0 ) {            // if found
    for(int j=i; j < size-1; ++j) {
      data[i] = data[i+1];  // shift all the elements by one
    }
    --size;                 // and reduce the array size
    return true;            // successfully removed the value
  }
  else {
    return false;           // could not find the value to remove
  }
}
```

Implementing complex data types is not so simple

```
int main(int argc, char** argv) {
  myArray<int> A;         // creating an instance of myArray
  A.insert(10);
  A.insert(20);
  myArray<int> B = A;     // copy the instance
  B.remove(10);
  if ( A.search(10) < 0 ) {
    std::cout << "Cannot find 10" << std::endl; // what would happen?
  }
  return 0;               // would to program terminate without errors?
}
```

Implementing complex data types is not so simple

```
int main(int argc, char** argv) {
  myArray<int> A;          // A is empty, A.data points an address x
  A.insert(10);            // A.data[0] = 10, A.size = 1
  A.insert(20);            // A.data[0] = 10, A.data[1] = 20, A.size = 2
  myArray<int> B = A;      // shallow copy, B.size == A.size, B.data == A.data
  B.remove(10);            // A.data[0] = 20, A size = 2 -- NOT GOOD
  if ( A.search(10) < 0 ) {
    std::cout << "Cannot find 10" << std::endl; // A.data is unwillingly modified
  }
  return 0;  // ERROR : both delete [] A.data and delete [] B.data is called
}
```

How to fix it

A naive fix : preventing object-to-object copy

```
template <class T>
class myArray {
protected:
   T *data;
   int size;
   int nalloc;
   myArray(myArray& a) {}; // do not allow copying object
public:
   myArray() {...};        // allow to create an object from scratch
```

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```

A complete fix

- `std::vector` does not suffer from these problems
- Implementing such a nicely-behaving complex object is NOT trivial
- Requires a deep understanding of C++ programming language

A practical advice in implementing a C++ class

- When there are already proven implementations, always utilize them
  - Standard Template Library for basic data structures
  - Boost Library for more sophisticated data types (e.g. Graphs)
  - `Eigen` package for matrix operations
  - ✓ *Always check the license carefully, especially if do not want to release your source code*

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    - ✓ *Always check the license carefully, especially if do not want to release your source code*
- If it is necessary to implement your own complex data types
    - Use STL (or other well-behaving) data types as member variables whenever possible
    - Keep the behavior simple and well-defined to reduce implementation overhead
    - However, if you spend your time to design your data type robust against many complex situations, your class will be very useful to others.

Next Lecture

Overview of elementary data structures

- Sorted array
- Linked list
- Binary search tree
- Hash table

Reading materials

- CLRS Chapter 10
- CLRS Chapter 11
- CLRS Chapter 12