# 2011 Winter BIOSTAT 615/815 Midterm

Thursday March 10th, 08:40AM-10:00AM

**NAME** : (                                        )

## Problem 1. True/False Questions (20 pts - 2pts each)

Write down **T** (True) or **F** (False) for each of the statement below.

1. (      ) In C++, `float` data type uses 4 bytes (32bits) and `double` uses 8 bytes (64bits) for most widely used platforms.
2. (      ) The number of values that `double` data type can represent is infinite.
3. (      ) The worst-case time complexity of binary search is $\Theta(\log n)$, assuming all elements are unique.
4. (      ) If all the elements in an array are single-digit integers, it is possible to sort the array in linear time complexity.
5. (      ) Quicksort has better worst-case time complexity than the MergeSort does.
6. (      ) When search is much more frequent than insertions or deletions in a container, `SortedArray` is a more efficient container than `List`.
7. (      ) The chance of key collison is equal or smaller in a chained hash than an open hash given a number of possible hash values (slots).
8. (      ) Dijkstra's algorithm is not guaranteed to work correctly if there are negatively weighted edges.
9. (      ) The fastest algorithm of matrix multiplication has $\Theta(n^2)$ time complexity.
10. (      ) If $\mathbf{u}$ and $\mathbf{u}$ are $n \times 1$ vectors, and $A$ and $B$ are $n \times n$ matrices, $(((\mathbf{u}^T A)B)\mathbf{v})$ has better time complexity than $((\mathbf{u}^T(AB))\mathbf{v})$.

## Problem 2. Short answer questions (20pts - 4pts each)

Write down the expected output for each program (4pts each).

(a)

```
#include <iostream>
int main() {
  std::cout << ( 1 + 2 * 3 ) << std::endl;
  return 0;
}
```

(b)

```
#include <iostream>
int main() {
  int a[3] = {1,2,3};
  int* b = &a[1];
  b[1] = 10;
  std::cout << a[0] << "," << a[1] << "," << a[2] << std::endl;
  return 0;
}
```

(c)
```cpp
#include <iostream>

int foo(int n) {
  if ( n == 0 )
     return 0;
  else
     return n + foo(n-1);
}

int main() {
  std::cout << foo(10) << std::endl;
}
```

(d)
```cpp
#include <iostream>
void foo(int n) {
   n = 0;
}
void bar(int& n) {
   n = 0;
}
int main() {
   int a = 3, b = 1;
   foo(a);
   bar(b);
   std::cout << a << "," << b << std::endl;
}
```

(e)
```cpp
#include <iostream>
#include <Eigen/Dense>
using namespace Eigen;
int main() {
  Matrix2d mat;
  mat << 1, 2,
         3, 4;
  Vector2d u(-1,1), v(2,0);
  std::cout << u.transpose() * mat * v << std::endl;
}
```

## Problem 3 - Long answer questions (30pts - 10pts each)

(a) Fill in the body of the `cumsum()` function so that the following properties hold upon the completion of the function.

$$dst[i] = \sum_{j=0}^{i} src[j]$$

for $i = 0, \cdots, n-1$. Note that the function must have $\Theta(n)$ time complexity

```
// function to calculate cumulative sums
// INPUT:
// n : number of elements in the arrays
// src is size-n array, containing integer values
// OUTPUT:
// dst is size-n array, uninitialized
// when the function ends, dst should contain
// dst[i] = \sum_{j=0}^i src[i]
void cumsum(int n, int* src, int* dst) {
// fill the blank below
























}
```

(b) Complete the three lines in the `search()` function of `MySortedArray`.

```
// "std::vector<T> data"  is assumed to be declared above as a member variable
// search() function returns the index of the element x,
// -1 is returned when the element x does not exist
template <class T>
int mySortedArray<T>::search(const T& x) {
  return search(x, 0, size-1);
}
template <class T>
int mySortedArray<T>::search(const T& x, int begin, int end) {
  if ( begin > end )
    return -1;
  else {
    int mid = (begin+end)/2;
    if ( data[mid] == x )
      return             // **** FILL IN LINE 1
    else if ( data[mid] < x )
      return             // **** FILL IN LINE 2
    else
      return             // **** FILL IN LINE 3
  }
}
```

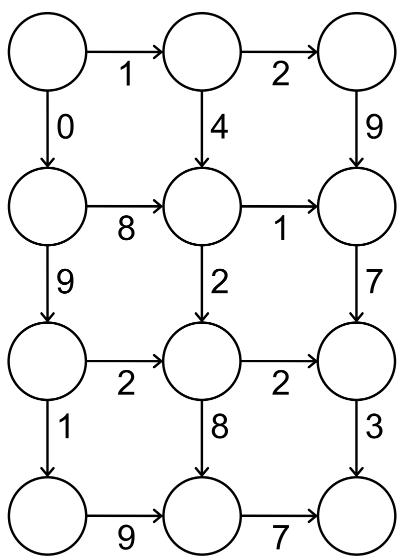
(c) Write down the expected output of the following code.

```
#include <iostream>
void towerOfHanoi(int n, int s, int i, int d) {
  std::cout << "towerOfHanoi called at n = " << n << std::endl;
  if ( n > 0 ) {
    towerOfHanoi(n-1,s,d,i);
```

```
    std::cout << "Disk " << n << " : " << s << " -> " << d << std::endl;
    towerOfHanoi(n-1,i,s,d);
  }
}
int main(int argc, char** argv) {
  towerOfHanoi(2, 1, 2, 3);
  return 0;
}
```

## Problem 4 - Dynamic Progrsamming (20pts - 5pts each)

Consider the following Manhattan tourist problem. The weight at each edge represents the time cost moving from one node to the other. Let $(i, j)$ represents the node at row $i$, column $j$.



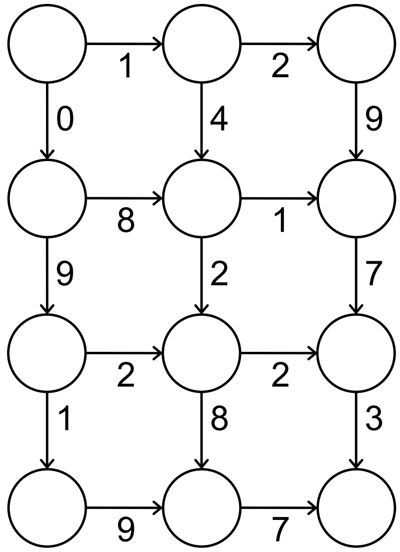

(a) How many possible paths do exist from node $(1, 1)$ (top-left) to node $(4, 3)$? (bottom-right)?

(b) Suppose that the following algorithm is used.

   (a) If current node is $(i, j)$, look at the next possible paths and choose the path with smaller weight

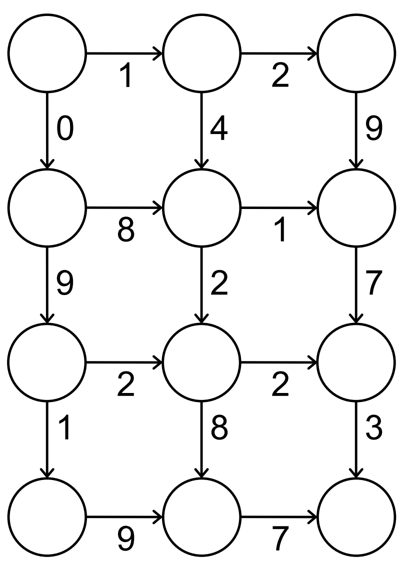
   (b) Repeat the above procedure until it reaches the destination

   What is the overall cost from $(1, 1)$ to $(4, 3)$ if traversed based on the algorithm above? Write the cost from the source to each node along the path.




(c) Let $d(i, j)$ be the optimal cost to reach to the node $(i, j)$. Write down a dynamic programming formulation to represent $d(i, j)$ as a function of optimal costs in the preceding nodes.

(d) Fill all the node with $d(i, j)$ in the figure below and highlight the path providing the optimal cost



## Problem 5. Hidden Markov Model (10pts)

Consider a $n$-state hidden Markov model $\lambda = \{\pi, A, B\}$ across $t$ time point where $\pi$ is prior distribution $A$ and $B$ represent transition and emission probability. (Same model as described in the class). Let $q_1, \cdots, q_T$ be the hidden state at time $1, \cdots, T$ and $o_1, \cdots, o_T$ be the observed outcomes.

(a) Given $q_1, \cdots, q_T$, are $o_1, \cdots, o_T$ conditionally independent of each other? Describe why briefly.

(b) Given $o_1, \cdots, o_T$, are $q_1, \cdots, q_T$ conditionally independent of each other? Describe why briefly.